R.P. 539
28-10-85

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Reprinted from FOREST SCIENCE
Vol. 26, No. 3, September 1980
pp. 369-373

Forest Sci., Vol. 26, No. 3, 1980, pp. 369–373

A Recursive Approach to Include Complex Demand Equations in Economic Harvest Scheduling

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ABSTRACT. The two current algorithms for harvest scheduling are based on a downward sloping linear demand curve that uses a simple demand for stumpage equation including only the quantity demanded and the own price as variables. This paper presents the economics and theoretical reasoning for including additional variables in the demand for stumpage equation. The linkage equation is redeveloped to include these variables over time. FOREST SCI. 26:369–373.

ADDITIONAL KEY WORDS. Forest economics, simulation, present net worth, econometrics.

THE SCHEDULING OF HARVEST VOLUMES OVER TIME using present net worth and present net benefit maximization,[1] which implicitly assumes a downward sloping linear demand schedule for stumpage,[2] was first presented by Walker (1971). Johnson and Scheurman (1977) formulated a nonlinear optimization approach to the solution. Although economic

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[1] Present net benefit maximization theoretically finds where consumer plus producer surplus is maximum.

[2] Higher order demand curves result in objective functions that are higher order than quadratic objective functions. Convexity problems then occur such that the algorithm is unable to determine the difference between local optima and the true optimum.

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considerations were primary in developing the method, the available literature has stressed largely the mathematical and operational characteristics of the method with little regard given the economic assumptions that are made in demand schedule specification. This paper addresses the suitability of the demand equation formulation used in the existing algorithms (Walker 1971, Tedder and others 1979) and presents an expansion to include more than one slope variable in the linkage equation.

CURRENT DEMAND EQUATION ASSUMPTION

The usual form of the demand equation is $Q_d = f(P)$ where Q_d = quantity demanded, P = own price. The basic equation is linear:

$$Q_d = z - R_1 P$$

$$P = \frac{z}{R_1} - \frac{Q_d}{R_1}$$

where Q_d = quantity demanded, z = intercept, R_1 = slope coefficient.

Let $\frac{z}{R_1} = \alpha$ and $\frac{1}{R_1} = B_1$; then $P = \alpha - B_1 Q_d$.

Following Tedder and others (1979) and Johnson and Scheurman (1977), the nonlinear objective function becomes:

$$\text{Maximize } \frac{\sum_{j=1}^{N} \left(\alpha - B_1 \sum_{i=-m}^{j-z} V_{ij} x_{ij}\right) \sum_{i=-m}^{j-z} V_{ij} x_{ij} - C_3 \sum_{i=-m}^{j-z} V_{ij} x_{ij}}{(1 + r)^{pj}} \tag{1}$$

Let

$$h_j = \sum_{i=-m}^{j-z} V_{ij} x_{ij}$$

where

x_{ij} = acres cut in period j from age class i,
V_{ij} = volume per acre, age class i, period j,
α = price intercept of the demand curve,
B_1 = slope coefficient for the demand curve,
C_3 = cost per unit of harvest,
r = annual discount rate, $r = D - 1$, $D = 1 + r$. If 1-year age classes are assumed, then D is the appropriate value. If age class intervals are assumed, then D^p is appropriate where p is the length of one period in years.
$-m$ = number of periods before the start of the planning horizon at which the oldest age class in the starting inventory was established.
z = minimum number of periods until regenerated stands are eligible for harvest,
N = number of periods in the planning horizon,
h_j = total harvest per period.

Equation (1) becomes

$$\text{Maximize } \sum_{j=1}^{N} \frac{(\alpha - B_1 h_j) h_j - C_3 h_j}{D^{pj}},$$

which becomes

$$\text{Maximize } \sum_{j=1}^{N} \frac{TR_j - TC_j}{D^{pj}} \tag{2}$$

where

TR_j = total revenue in period j,
TC_j = total cost in period j.

The specific condition defining the mathematical relation that will locate the harvest maximizing present net worth is found from (2):

$$\frac{\partial TR}{\partial x_{ij}} = \sum_{j=1}^{N} \frac{(\alpha - 2B_1 h_j)V_{ij}}{D^{\nu j}}$$

and

$$\frac{\partial TC}{\partial x_{ij}} = \sum_{j=1}^{N} \frac{C_3 V_{ij}}{D^{\nu j}}$$

where

$\frac{\partial TR_j}{\partial x_{ij}}$ and $\frac{\partial TC_j}{\partial x_{ij}}$ are the marginal revenue and cost due to a unit change in acres harvested from age class i in period j. Similar relationships hold for $j = i, \ldots, N$.

Therefore $(\alpha - 2B_1 h_j)V_{ij} - C_3 V_{ij}$ becomes the Kuhn-Tucker condition allowing development of the linkage equation. Under the conditions stated by Schmidt and others (1980), the final linkage equation becomes

$$\frac{(\alpha - 2B_1 h_j)V_{ij} - C_3 V_{ij}}{D^{\nu j}} = \frac{(\alpha - 2B_1 h_{j+1})V_{ij+1} - C_3 V_{ij+1}}{D^{\nu j+1}}.$$

Solving for h_{j+1}, the equation becomes

$$h_{j+1} = \frac{C_4}{2B_1}\left[1 - D^{\nu}\left(\frac{V_{ij}}{V_{ij+1}}\right)\right] + D^{\nu}\left(\frac{V_{ij}}{V_{ij+1}}\right)h_j$$

where

$$C_4 = \alpha - C_3.$$

Schmidt and others (1980) indicate that the algorithm is insensitive to other demand shifters. This, of course, is both a theoretical and an operational problem. The usefulness of the algorithm seems limited to large areas of forest land, such as National Forests, which are able to affect stumpage when their harvest volumes are adjusted.

ALGORITHM EXPANSION

Adams and Haynes (1979) have developed a reactive econometric model in which the regional demand for National Forest stumpage may be derived. The resulting equation has more variables than can be included in the existing algorithm. However, we can develop an algorithm that provides for the inclusion of these variables. Suppose that demand is given by

$$Q_{dj} = f(P_j, P_{sj}, A_{sj})$$

where

Q_{dj} = quantity demanded in period j,
P_j = own price in period j,
P_{sj} = substitute price in period j,
A_{sj} = demand shifter in period j.

Assume that f is linear so that

$$Q_{dj} = \alpha - B_1 P_j + B_2 P_{sj} + B_3 A_{sj}$$

and

$$P_j = \frac{\alpha}{B_1} - \frac{Q_{dj}}{B_1} + \frac{B_2}{B_1} P_{sj} + \frac{B_3}{B_1} A_{sj}$$

where

B_2 = slope coefficient associated with variable P_{sj},

B_3 = slope coefficient associated with variable A_{sj},
Q_{dj} = harvest in period j (h_j).

Forming the total revenue portion of the objective function, (2) becomes

$$\sum_{j=1}^{N} TR_j = \sum_{j=1}^{N} \left(\frac{\alpha}{B_1} - \frac{h_j}{B_1} + \frac{B_2}{B_1}P_{sj} + \frac{B_3}{B_1}A_{sj}\right)h_j = TR$$

and

$$\sum_{j=1}^{N} TC_j = \sum_{j=1}^{N} C_3 h_j = TC.$$

Then the objective function becomes

$$\text{Maximize } \sum_{j=1}^{N} \frac{TR_j - TC_j}{D^{pj}} = \sum_{j=1}^{N} \frac{\left(\frac{\alpha}{B_1} - \frac{h_j}{B_1} + \frac{B_2}{B_1}P_{sj} + \frac{B_3}{B_1}A_{sj}\right)h_j - C_3 h_j}{D^{pj}}.$$

By taking the derivative of the objective function with respect to x_{ij}, we get

$$\frac{\partial TR - TC}{\partial x_{ij}} = \sum_{j=1}^{N} \frac{\left(\frac{\alpha}{B_1}V_{ij} - \frac{2h_jV_{ij}}{B_1} + \frac{B_2V_{ij}P_{sj}}{B_1} + \frac{B_3V_{ij}A_{sj}}{B_1}\right) - C_3V_{ij}}{D^{pj}}$$

$$= \sum_{j=1}^{N} \frac{\left(\frac{\alpha}{B_1} - \frac{2h_j}{B_1} + \frac{B_2P_{sj}}{B_1} + \frac{B_3A_{sj}}{B_1}\right)V_{ij} - C_3V_{ij}}{D^{pj}}. \tag{3}$$

The linkage equation is formed by again setting (3) equal to its counterpart in the next time period. It becomes

$$\frac{\left(\frac{\alpha}{B_1} - \frac{2h_j}{B_1} + \frac{B_2P_{sj}}{B_1} + \frac{B_3A_{sj}}{B_1}\right)V_{ij} - C_3V_{ij}}{D^{pj}}$$

$$= \frac{\left(\frac{\alpha}{B_1} - \frac{2h_{j+1}}{B_1} + \frac{B_2P_{sj+1}}{B_1} + \frac{B_3A_{sj+1}}{B_1}\right)V_{ij+1} - C_3V_{ij+1}}{D^{pj+1}} \tag{4}$$

where P_{sj+1} and A_{sj+1} are functions of h_j, P_{sj}, A_{sj}, P_j.

Solving for h_{j+1}, (4) becomes

$$h_{j+1} = \left(D^p\frac{V_{ij}}{V_{ij+1}}\right)h_j + \frac{\left[D^p\left(\frac{\alpha}{B_1} - C_3 + \frac{B_2}{B_1}P_{sj} + \frac{B_3}{B_1}A_{sj}\right)\right]\frac{V_{ij}}{V_{ij+1}}}{2B_1}$$

$$- \frac{\left[\left(\frac{\alpha}{B_1} - C_3\right) + \frac{B_2}{B_1}P_{sj+1} + \frac{B_3}{B_1}A_{sj+1}\right]}{2B_1}. \tag{5}$$

Similarly, the cost function may be expanded to include additional variables. Note that each demand variable is subscripted by period to account for time. It has been suggested that the demand equation can be collapsed into two variables by combining the slope shifters, A_{sj} and P_{sj}, into the intercept, thereby reverting to the original formula. However, the original formula is not correct when one considers the dynamic behavior of supply and demand analysis. The model determines harvest volumes on an iterative basis beginning with the initial guess in the first period. From that value the second-period harvest level is determined, and so on, period by period. Unless the initial guess is such that equilibrium conditions are met (Sessions 1977), the process begins again and converges on the optimal solution through a binary search (Johnson and Tedder[3]). Because harvest volumes through time are unknown, it is impossible to estimate the periodic demand curves (P_{sj+1} and A_{sj+1} are functions of h_j, P_{sj}, A_{sj}, P_j) and to collapse the equation into a two-variable linear function. As the variables are determined recursively in the expanded demand equation, it becomes clear that the solution process is dynamic.

CONCLUSIONS

The theory presented here allows for the inclusion of more rigorously developed demand and supply equations in the economic harvest scheduling techniques. It is doubtful that scheduling with this method is useful for small forests because they are not able to effect a change of price through different levels of volume harvested. Large forest holdings such as the National Forest system could fall into an imperfect market structure that would allow the use of the method.

The development of information in this paper can lead to a more precise determination of the economic harvest scheduling process involving the use of equations incorporating the most current regional stumpage demand.

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