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Dynamic Programming for Optimization of Timber Production and Grazing in Ponderosa Pine

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ABSTRACT. Dynamic programming procedures are presented for optimizing thinning and rotation of even-aged ponderosa pine by using the four descriptors: age, basal area, number of trees, and time since thinning. Because both timber yield and grazing yield are functions of stand density, the two outputs—forage and timber—can both be optimized. The soil expectation values for single and joint production are derived and compared, and the impact of dynamic changes in relative price of the two products over the rotation is shown. Depending on relative prices and discount rate, the maximum soil expectations will be provided by timber alone, grazing alone, or an optimal schedule of joint production. Impacts of relative costs and values of the two outputs on management are discussed. FOREST SCI. 28:517–526.

ADDITIONAL KEY WORDS. *Pinus ponderosa,* multiple products, forest-range management, thinning, resource economics, operations research.

THE APPLICATION OF DYNAMIC PROGRAMMING (DP) for the simultaneous determination of thinning intensity and rotation has been discussed by Hann and Brodie (1980). A typical DP formulation (e.g., Brodie and Kao 1979) maximizes the cumulative net present value of timber harvested over discrete levels of each of several state descriptors at each time stage defined in the network. Although DP formulations are becoming increasingly complex in terms of number of state descriptors and treatment options (e.g., Kao 1980), objective functions usually have been based on the value of timber only. In this paper, we demonstrate the use of dynamic programming for the simultaneous determination of thinning intensity and rotation for ponderosa pine (*Pinus ponderosa* Laws.) when two outputs, timber and forage, are considered in the objective function. Joint production is contrasted with production of each product independently, and the impact of changing relative values is discussed and demonstrated.

Burde (1974) conducted a case study of timber and forage production for a hypothetical ponderosa pine forest in Arizona. He concluded that the forest should be managed under the concept of dominant use with commercial rangeland managed for forage production, commercial forest land managed for timber production, and lands submarginal for either purpose managed for yet other uses. The problem we address is that of determining the thinning regime that jointly maximizes the returns from both grazing and timber harvest on the same unit of

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land. Forage production decreases with increasing stand density (Jameson 1967), while timber quantity and quality also vary with density. When the two outputs are considered jointly, there are trade-offs between the level of production of each; optimal production levels of both outputs depend on the relative value of each.

Dynamic programming can be used to solve this problem if the production level of both outputs can be estimated from the same set of state descriptors and if the values of both outputs can be measured on a common scale. In our formulation, forage production is based on stand basal area; timber production is based on stand basal area, number of trees, and time since last thinning; and the joint maximization of the present net worth (PNW) of both outputs is used as the objective criterion.

THE TIMBER GROWTH AND VALUATION MODELS

The timber growth model, identified by the acronym PPINE (Hann 1980), is a whole-stand, diameter-class model that simulates 5-year growth of even-aged ponderosa pine stands in northern Arizona on the basis of site index, number of years since last thinning, and the number of trees per acre in 1-inch diameter classes. With this model, we explicitly incorporate quality premium information by assigning higher stumpage values to larger trees. The contribution from timber harvest to the objective function in year t, $\text{PNW}_W{}^{(t)}$, is calculated as

$$\text{PNW}_W{}^{(t)} = \frac{\sum_{D=1}^{40} S_D T_D - C_1}{(1 + i)^t} \tag{1}$$

where S_D is the stumpage price, T_D is the number of trees harvested by diameter class (D), C_1 represents a fixed entry cost, and i is the interest rate.

THE GRAZING PRODUCTION AND VALUATION MODELS

The grazing model was derived from reports of studies conducted on the Wild Bill Range in northern Arizona. Annual forage production (H), in pounds per acre, is computed by the following equation (Jameson 1967):

$$H = 672 - \left[628 \cdot (1 - e^{-0.048 \cdot BA})^{1.25}\right] \tag{2}$$

where BA is the average of stand basal area, in square feet per acre, before and after each 5-year cycle of stand growth. Assuming a forage utilization rate (U) of 30 percent (Pearson 1973) and an *in vitro* digestibility (d) of 54 percent (Pearson 1964), annual beef gain (G), in pounds per acre, is computed from the equation derived from Pearson (1972, 1973):

$$\begin{aligned} G &= 0.257 \cdot HdU - 1.013 \\ &= 0.0416 \cdot H - 1.013. \end{aligned} \tag{3}$$

The contribution from grazing to the objective function in year t, $\text{PNW}_F{}^{(t)}$, is calculated as

$$\text{PNW}_F{}^{(t)} = \sum_{j=1}^{I} \left[\sum_{k=1}^{5} \frac{[B \cdot G_j - C_2]}{(1 + i)^{t-5(I)+5(j-1)+k}} \right] \tag{4}$$

where B is beef price per pound; C_2 is a fixed annual grazing cost; and I is the number of 5-year growth periods per time step in the network. The term inside the brackets is a summation of discounted revenue over the 5 years in any one growth period. This term is then summed over I growth periods. If the value within the brackets is negative for any j, then no grazing is assumed for that growth period.

Structure of the Dynamic Programming Algorithm

The structure of the DP algorithm is essentially the same as that of a model by Brodie and Kao (1979), a forward-solution method that employs basal area (BA), number of trees (N), and stand age (A) as discrete state descriptors. However, an extra descriptor was added to account for the relationship between stand growth and time since last thinning (L). Because of this addition and because each stand is characterized by numbers of trees in each of 40 1-inch diameter classes, computer storage space quickly became limiting. This problem was solved by using random access files and by "packing" stand information into 25 computer words per stand. Efficient search strategies were possible because all stands thinned at any stage have a common value of L and because the density of the network in the L-dimension is less than would be expected for a descriptor such as BA.

At each node in the network, we stored the following information:

A. Residual number of trees by 1-inch (2.54-cm) diameter class for 40 diameter classes.
B. Volume, basal area, and number of trees (cut and residual).
C. Cumulative value of the objective function.
D. Contribution to the objective function in the current stage from grazing.
E. Where the stand came from (BA,N,L,A) in the last stage.
F. Maximum diameter achieved by the stand.
G. Thinning type used in current stage.

To solve the DP problem, we defined the optimal-value PNW function, $f_t(BA,N,L)$, as the value of the PNW path from regeneration to age t, basal area BA, number of trees N, and time since last thinning L for the stand:

$$f_t(BA,N,L) = \underset{\{ba,n,l\}}{\text{Max}} \left[\text{PNW}_W{}^{(t)} + \text{PNW}_F{}^{(t)}\right] + f_{t-5(I)}(ba,n,l) \qquad (5)$$

where $\{ba,n,l\}$ is the set of all feasible nodes (basal area, number of trees, and time since last thinning) at age $t - 5(I)$ from which the current node described by (BA,N,L) can be reached. The starting condition is

$$f_{30}(BA,N,L) = R$$

where R is the present net worth of timber and forage production before age 30 (the first candidate age for commercial thinning). The recursion of equation (5) can be terminated when the soil expectation (SE), or capitalized value, of $f_t(0,0,0)$ declines, if the objective function is known to be convex, or after a pre-set number of stages, if convexity cannot be assumed.

Demonstration of Timber and Grazing Optimization

To demonstrate the joint optimization of timber and grazing, we will show optimal treatment regimes determined for five different cases:

A. Grazing only.
B. Timber only.
C. Grazing and timber at current prices.
D. Grazing and timber with timber stumpage increasing 1 percent per year.
E. Grazing and timber with beef prices increasing 1 percent per year.

TABLE 1. Base stumpage schedule for timber.

Dbh class (inches)	Price per tree	Dbh class (inches)	Price per tree
	dollars		*dollars*
1–6	0.00	19	148.80
7	.20	20	177.04
8	.81	21	216.84
9	3.85	22	260.04
10	8.55	23	301.79
11	14.77	24	346.41
12	22.49	25	409.38
13	33.35	26	476.71
14	45.86	27	513.84
15	62.60	28	550.52
16	81.33	29	589.04
17	101.09	30	627.10
18	122.87	31	666.09

We chose state descriptor intervals of $A = 20$ years, $BA = 15$ square feet per acre, $N = 15$ trees per acre, and $L = 20$ years. In preliminary analyses, these intervals provided adequate sensitivity without unduly increasing computation time. Thinning was constrained so that at least 10 percent of gross total cubic foot stocking was removed. For any given thinning, a constant proportion of trees was removed from each diameter class.

In all cases, a real interest rate of 3 percent was used. Base stumpage prices for timber are shown in Table 1. A fixed thinning entry cost of $5/acre was assumed. Base price for beef is $0.78/lb,[1] and a fixed grazing cost of $2/acre/year was used.

In Case D, timber stumpage was compounded to the time of thinning or final harvest. In Case E, beef price was compounded to the middle of each 5-year period of stand growth so as to coincide in time with the estimate of average forage production.

Initial stand conditions at age 30 were obtained by using the PPINE model to simulate the development of a site 88 stand with an assumed structure of 200 1-inch trees and 140 2-inch trees at age 15. A regeneration cost of $250/acre was assumed. Initial values for grazing were computed as shown in Table 2.

The results of the five economic optimizations are presented in Tables 3A–E in detail and summarized in Table 4. Case C (joint production) provides an *SE* that is 37 percent higher than Case A (grazing only) and 54 percent higher than Case B (timber only), with an interesting comparison of the timing of the two outputs in Case C. Forage production dominates in the first 30 years, effectively neutralizing the stand regeneration cost so that a shorter rotation (in comparison with Case B) is indicated. This is evidenced by the fact (not shown) that although the PNW for Case C peaked at 110 years, *SE* peaked at 90 years. Virtually no grazing occurs after age 30, when timber revenues dominate the objective function.

The mean annual wood production (MAI) of Case C is only slightly less than that of Case B, and although the shorter rotation of Case C produced timber of

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[1] August 1980 price for feeder calves, Wall Street Journal. Pearson (1972, 1973) used yearling weight gains in his work. For the purpose of analysis, it is the price relative to timber that influences results.

TABLE 2. Computation of initial values per acre (ha) for grazing.

			Constant relative beef price		Increasing relative beef price[b]	
Age (years)	Average basal area	Annual forage production	Annual net revenue	PNW[a]	Annual net revenue	PNW[a]
			dollars			
	ft² *(m²)*	*lb* *(kg)*	*$/acre* *($/ha)*	*$/acre* *($/ha)*	*$/acre* *($/ha)*	*$/acre* *($/ha)*
0–5	0 (0)	672.0 (753.2)	19.03 (47.02)	87.16 (215.38)	19.58 (48.38)	89.67 (221.58)
5–10	0 (0)	672.0 (753.2)	19.03 (47.02)	75.19 (185.80)	20.65 (51.03)	81.60 (201.64)
10–15	2.28 (0.52)	635.1 (711.8)	17.83 (44.06)	60.77 (150.17)	20.38 (50.36)	69.44 (171.59)
15–20	9.24 (2.12)	498.1 (558.3)	13.39 (33.09)	39.35 (97.24)	16.34 (40.38)	48.03 (118.69)
20–25	21.79 (5.00)	306.5 (343.5)	7.16 (17.69)	18.16 (44.88)	9.52 (23.52)	24.13 (59.63)
25–30	39.64 (9.10)	158.8 (178.0)	2.37 (5.86)	5.18 (12.80)	3.77 (9.32)	8.24 (20.36)
Total	—	—	—	285.81 (706.27)	—	321.11 (793.49)

[a] Discount rate is 3 percent per year.

[b] Rate of price increase is 1 percent per year.

smaller diameter at final harvest, quadratic mean diameter at comparable ages is greater in Case C because of lower residual densities.

Increasing timber stumpage relative to beef price (Case D) leads to increases in *SE*, rotation, quadratic mean diameter at harvest, and MAI in comparison with Cases B and C. Higher densities are maintained early in the rotation to make the most of high late-rotation stumpage prices, which apparently offset possible value gains from maintaining lower densities.

Increasing beef price relative to timber stumpage (Case E) leads to increases in *SE* and quadratic mean diameter at harvest, but also to a lower MAI in comparison with Case C. Rotation is unaffected. Very low densities are maintained late in the rotation, and this is the only case where grazing significantly contributes to *SE* late in the rotation.

JOINT PRODUCTION COST AND REVENUE IMPACTS

Brodie and others (1978) reviewed the economic impacts of various costs of timber production with dynamic density control through thinning during the ro-

TABLE 3A. Management for grazing only on ponderosa pine site 88, discount rate = 3 percent.

Annual forage production = 672.0 lb/acre (753.2 kg/ha)
Annual beef gain = 26.97 lb/acre (30.22 kg/ha)
Annual net revenue = $19.03/acre ($47.02/ha)
Soil expectation = $634.33/acre ($1,567.46/ha)

TABLE 3B. Optimal management for timber only on ponderosa pine site 88, discount rate = 3 percent.[a]

	Residual per acre (ha)			Harvest per acre (ha)			Grazing PNW contribution per acre	Cumulative PNW per acre	Quadratic mean dbh
Age (years)	Trees	Basal area	Volume	Trees	Basal area	Volume			
	number	*ft²* *(m²)*	*ft³* *(m³)*	*number*	*ft²* *(m²)*	*ft³* *(m³)*	*dollars*	*dollars*	*inches* *(cm)*
30	300	44.37	269.5	35.5	5.25	31.9	0	−251.06	5.2
	(741.3)	(10.19)	(18.9)	(87.7)	(1.21)	(2.2)			(13.2)
50	210.0	85.32	922.5	84.4	34.29	370.7	0	−136.52	8.6
	(518.9)	(19.59)	(64.6)	(208.6)	(7.87)	(25.9)			(21.8)
70	105.0	69.28	994.9	100.8	66.52	955.2	0	135.46	11.0
	(259.5)	(15.90)	(69.6)	(249.1)	(15.27)	(66.8)			(27.9)
90	30.0	30.41	560.7	72.8	73.80	1,360.5	0	399.57	13.6
	(74.1)	(6.98)	(39.2)	(179.9)	(16.94)	(95.2)			(34.5)
110	0.0	0.0	0.0	29.3	48.35	1,100.8	0	540.31	17.4
	(0.0)	(0.0)	(0.0)	(72.4)	(11.10)	(77.0)			(44.2)
Total	—	—	—	—	—	3,819.1	—	—	—
						(267.2)			

[a] Rotation = 110 years. MAI = 34.72 ft³/acre/yr (2.43 m³/ha/yr). Soil expectation = $562.07/acre ($1,388.90/ha).

TABLE 3C. Optimal management for joint production of timber and grazing at current prices on ponderosa pine site 88, discount rate = 3 percent.[a]

	Residual per acre (ha)			Harvest per acre (ha)			Grazing PNW contribution per acre	Cumulative PNW per acre	Quadratic mean dbh
Age (years)	Trees	Basal area	Volume	Trees	Basal area	Volume			
	number	*ft²* *(m²)*	*ft³* *(m³)*	*number*	*ft²* *(m²)*	*ft³* *(m³)*	*dollars*	*dollars*	*inches* *(cm)*
30	285.0	42.15	256.1	50.5	7.47	45.5	285.81	35.13	5.2
	(704.2)	(9.68)	(17.9)	(124.8)	(1.71)	(3.2)			(13.2)
50	165.0	68.33	741.7	114.7	47.49	515.5	1.32	199.38	8.7
	(407.7)	(15.69)	(51.9)	(283.4)	(10.90)	(36.1)			(22.1)
70	75.0	53.94	790.0	86.7	62.37	913.5	0.00	474.26	11.5
	(183.3)	(12.38)	(55.3)	(214.2)	(14.32)	(63.9)			(29.2)
90	0.0	0.0	0.0	73.4	84.78	1,611.6	0.11	806.90	14.6
	(0.0)	(0.0)	(0.0)	(181.4)	(19.46)	(112.8)			(37.1)
Total	—	—	—	—	—	3,086.0	—	—	—
						(215.9)			

[a] Rotation = 90 years. MAI = 34.29 ft³/acre/yr (2.40 m³/ha/yr). Soil expectation = $867.57/acre ($2,143.77/ha).

TABLE 3D. Optimal management for joint production of timber and grazing with relative timber price increasing at 1 percent per year on ponderosa pine site 88, discount rate = 3 percent.[a]

Age (years)	Residual per acre (ha)			Harvest per acre (ha)			Grazing PNW contribution per acre	Cumulative PNW per acre	Quadratic mean dbh
	Trees	Basal area	Volume	Trees	Basal area	Volume			
	number	*ft^2 (m^2)*	*ft^3 (m^3)*	*number*	*ft^2 (m^2)*	*ft^3 (m^3)*	*dollars*	*dollars*	*inches (cm)*
30	300.0 (741.3)	44.37 (11.19)	269.5 (18.9)	35.5 (87.7)	5.25 (1.21)	31.9 (2.2)	285.81	35.06	5.2 (13.2)
50	255.0 (630.1)	103.61 (23.79)	1,120.1 (78.4)	39.4 (97.4)	16.00 (3.67)	173.0 (12.1)	0.88	123.58	8.6 (21.8)
70	135.0 (333.6)	83.53 (19.18)	1,210.6 (84.7)	114.9 (283.9)	71.10 (16.32)	1,030.4 (72.1)	0.00	674.49	10.7 (27.2)
90	75.0 (183.3)	68.61 (15.75)	1,261.7 (88.3)	57.2 (141.3)	52.31 (12.01)	961.9 (67.3)	0.00	1,100.58	13.0 (33.0)
110	30.0 (74.1)	39.24 (9.01)	863.0 (60.4)	43.3 (107.0)	56.67 (13.01)	1,246.3 (87.2)	0.00	1,529.67	15.5 (39.4)
130	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	29.3 (72.4)	56.09 (12.88)	1,441.5 (100.9)	0.76	1,904.73	18.7 (47.5)
Total	—	—	—	—	—	4,885.0 (341.8)	—	—	—

[a] Rotation = 130 years. MAI = 37.58 ft³/acre/yr (2.63 m³/ha/yr). Soil expectation = $1,946.46/acre ($4,809.71/ha).

TABLE 3E. Optimal management for joint production of timber and grazing with relative beef price increasing at 1 percent per year on ponderosa pine site 88, discount rate = 3 percent.[a]

Age (years)	Residual per acre (ha)			Harvest per acre (ha)			Grazing PNW contribution per acre	Cumulative PNW per acre	Quadratic mean dbh
	Trees	Basal area	Volume	Trees	Basal area	Volume			
	number	*ft^2 (m^2)*	*ft^3 (m^3)*	*number*	*ft^2 (m^2)*	*ft^3 (m^3)*	*dollars*	*dollars*	*inches (cm)*
30	300.0 (741.3)	44.37 (10.19)	269.5 (18.9)	35.5 (87.7)	5.25 (1.21)	31.9 (2.2)	321.11	70.02	5.2 (13.2)
50	135.0 (333.6)	54.85 (12.59)	593.0 (41.5)	159.4 (393.9)	64.76 (14.87)	700.1 (49.0)	2.66	290.01	8.6 (21.8)
70	30.0 (74.1)	22.71 (5.21)	344.7 (24.1)	102.3 (252.8)	77.46 (17.78)	1,175.9 (82.3)	1.83	648.02	11.8 (30.0)
90	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	29.4 (72.6)	41.31 (9.48)	822.9 (57.6)	22.01	855.23	16.1 (40.9)
Total	—	—	—	—	—	2,730.8 (191.1)	—	—	—

[a] Rotation = 90 years. MAI = 30.34 ft³/acre/yr (2.12 m³/ha/yr). Soil expectation = $919.53/acre ($2,272.16/ha).

TABLE 4. Summary results of optimization examples for ponderosa pine site 88.

Case	Soil expectation (3 percent)	Rotation	Quadratic mean dbh at final harvest	MAI
	dollars/acre *(dollars/ha)*	*years*	*inches* *(cm)*	*ft³/acre/yr* *(m³/ha/yr)*
A. Grazing (G)	634.33 (1,567.46)	—	—	0.0 (0.0)
B. Timber (T)	562.07 (1,388.90)	110	17.4 (44.2)	34.72 (2.43)
C. G and T	867.57 (2,143.77)	90	14.6 (37.1)	34.29 (2.40)
D. G and T, T price increasing	1,946.46 (4,809.71)	130	18.7 (47.5)	37.58 (2.63)
E. G and T, G price increasing	919.53 (2,272.16)	90	16.1 (40.9)	30.34 (2.12)

tation. Also of interest are observations on the role that joint consideration of grazing imposes. The physical relation between grazing and timber production in our example is one of competitive outputs; however, except for a rotation impact, our examples with grazing and timber and timber only at constant relative values show identical thinning regimes for fixed rotation length. This comparison of regimes for a fixed rotation, although not shown, can be drawn from analysis of intermediate solutions. Grazing results in the shortening of rotations because the highest grazing revenues occur early in the rotation and offset the lengthened rotations caused by regeneration cost. At much lower relative values of timber to grazing (or higher relative values of grazing to timber), the optimum density of timber throughout the rotation is reduced. Sensitivity analysis through lowering the relative value of timber would result in shorter rotations and heavier thinning, with the grazing-only alternative eventually becoming optimum. But, if the relative value of timber is increased, grazing will still remain in the optimal solution because the revenues occur during the early phase of the rotation when timber density is not controlled within a range highly competitive with grazing. Higher discount rates will affect timber revenues more heavily than grazing revenues because of the timing of each, shifting solutions toward shorter rotations, lower soil expectation, and a higher proportion of value attributable to grazing.

In the case of increasing grazing values, our analysis is for a single rotation. The soil expectation is achieved by an infinite series transformation that assumes no change in relative values in later rotations. A switch to grazing-only management may be indicated at the end of the first rotation; the timing and desirability of this shift could be analyzed with dynamic programming as suggested by Murphy and others (1977). We believe this question to be of theoretical interest to managers with stands late in rotation; it could best be analyzed with the present algorithm and sensitivity analysis of relative prices. The cost and revenue structures of both timber and grazing shift over time, both relatively and absolutely, and an optimization framework of analysis provides important insights into production opportunities.

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[2] For example, if an equation that relates water production to basal area were used, optimization could take place over three outputs.

We did not model the effects of physical interactions between the two production activities. Mechanical damage to tree seedlings from grazing is an example of such an interaction. Regeneration delay, exclusion of grazing for several years early in each rotation, and increased seedling protection are alternatives that could be considered. Regeneration delay would be optimal only if the value of a site were greater for forage production alone than for joint production. In terms of the model, the effect of the other two alternatives would be to increase regeneration cost, and the choice between them would depend on the price of forage revenue foregone relative to the cost of seedling protection. These activities could alter the optimal regimes indicated by our examples.

CONCLUSIONS

We have demonstrated the use of dynamic programming for the simultaneous determination of thinning intensity and rotation for ponderosa pine when two outputs, timber and forage, are considered. The methodology described could be applied to any number of outputs that can be estimated from the set of state descriptors.[2]

A four-descriptor dynamic programming algorithm was constructed for the solution of the problem. Efficient storage of stand information and the special nature of one of the descriptors allowed us to employ a diameter-class stand growth model. This, in turn, allowed more realistic representation of the stands and of the effect of quality premiums.

Joint production of timber and forage has a higher soil expectation (*SE*) than either produced alone. Most grazing occurs in the first 30 years of the rotation and tends to shorten rotations by offsetting costs of stand regeneration. Increases in relative stumpage prices tend to lengthen rotations and to increase mean annual wood production (MAI) and average diameter at harvest. The solution is less sensitive to increases in relative beef prices, except late in the rotation when a very low stand density is indicated.

Contrary to the dominant-use concept of Burde (1974), our results suggest that joint production strategies that emphasize different outputs at different stages of the rotation can be attractive alternatives to strategies calling for production of a single product.

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