

RESPONSES TO RISK IN WEED CONTROL DECISIONS UNDER EXPECTED PROFIT MAXIMISATION

D. J. Pannell*

Risk is an important characteristic of decisions about weed control in crops. In this paper it is shown that risk can affect weed control decisions even if the objective of the decision maker is to maximise expected profits: that is, even if the decision maker is 'risk-neutral' in the usual economic sense. This is shown for two decision frameworks: the optimal rate approach and the economic threshold approach. Empirical results are presented for control of ryegrass in wheat in Western Australia. It is found that, in general, risk reduces the optimal level of herbicide use under expected profit maximisation. Although individual sources of risk have a small impact on the optimal decision rules, combinations of uncertain variables can have a relatively large effect.

Introduction

In many studies of the economics of pest or weed control, the assumed objective of decision makers is maximisation of expected profit (e.g. Marra and Carlson, 1983; Moffitt *et al.*, 1984; Taylor and Burt, 1984; Gold and Sutton, 1986; Johnston and Price, 1986; Zacharias *et al.*, 1986). There is evidence that, in some circumstances, optimal pesticide decisions under expected profit maximisation differ little from decisions made under risk aversion (Webster, 1977; Thornton, 1984). Of those studies which assume risk-neutrality, the majority adopt a deterministic decision framework. This approach can sometimes be defended on the basis that the decision which maximises expected profit in a stochastic framework corresponds to the profit-maximising decision in a deterministic framework using expected values of parameters. This is why expected profit-maximisers are often referred to as 'risk-neutral'.

Nevertheless, there are several ways in which risk can affect the decisions of individuals whose objective is to maximise expected profit. Tisdell (1986) showed that uncertainty about a parameter value can affect the optimal level of

* Mr David Pannell is a PhD candidate in the University of Western Australia, currently a Research Economist at the Dryland Research Institute, Department of Agriculture, Merredin W.A. 6415, Australia. This paper is a revised version of the winning submission for the Agricultural Economic Society's Prize Essay Competition for 1989. The research was conducted whilst on study leave at the University of Western Australia. Bob Lindner is thanked for his help and support, Rob Fraser for help with the mathematical derivations, and the Society's Editorial Committee for constructive comments on the manuscript. The author is grateful to the Department of Agriculture for granting the study leave.

pest control by affecting expected profit. This is due to non-linearity in the model. He argued that 'in many cases the expected level of application is greater under uncertainty than under full information but . . . this depends on convexity conditions of relevant functions' (p. 161) and that 'convexity conditions may sometimes be such as to give rise to the opposite consequence' (p. 159).

He did not discuss which parameters are likely to increase and which to decrease treatment levels under uncertainty. Auld and Tisdell (1986, 1987, 1988) showed that, because of convexity of the relationship between weed density and crop yield, uncertainty about weed density reduces expected yield loss. Auld and Tisdell (1987) argued that this increases the threshold weed density for herbicide use, reducing the expected level of herbicide use. They noted that this does not seem consistent with comments in the literature that risk increases pesticide use. They attributed the difference to the influence of risk aversion dominating the effect of risk on expected profit.

Plant (1986) conducted a similar study for insect pests and found that uncertainty about pest mortality increased the economic threshold. Although he interpreted this as implying an increase in pesticide use, this is incorrect. Even with Plant's little-used definition of the threshold, a higher threshold implies lower pesticide use, consistent with Auld and Tisdell's (1987) argument.

Another circumstance where risk can affect the decisions of 'risk-neutral' decision makers is where the problem is dynamic (Antle, 1983). Zacharias *et al.* (1986) tested this hypothesis in their dynamic programming study of the soybean cyst nematode. They found modest support for the hypothesis, with very small differences between the results of their deterministic and stochastic models.

A third possibility is where the decision maker is subject to a progressive marginal taxation rate. Taylor (1986) showed that the effect of this on decision making is essentially the same as the effect of risk aversion; it makes the decision maker behave in a more risk-averse manner than he otherwise would have.

This paper is an examination of the effects of risk on weed control decisions made by 'risk-neutral' farmers. Attention is focused on effects due to non-linearities in the response model rather than dynamic or tax-induced effects. Sources of risk which affect expected profit are identified and analysed in the context of a theoretical response model for herbicide application. The direction of response to different sources of risk are derived using the theoretical model, and the magnitudes of the responses are estimated for a particular empirical example: control of ryegrass (*lolium rigidum*) in wheat in Western Australia.

Response Model

Lichtenberg and Zilberman (1986) showed that yield response to pesticides must be represented as an indirect response; pesticides kill pests and it is the removal of pests which increases yield. Use of a response model which does not recognise this two-stage process would lead to biased predictions of response and erroneous conclusions about the optimal pesticide strategy.

In this paper, crop yield (Y) is represented using the following general form:

$$Y = Y_0 [1 - D(W)] \quad (1)$$

where Y_0 is production with no weeds present and D is the damage function representing the proportion of production lost at weed density W .

Cousens (1985) conducted tests of a wide range of functional forms for the damage function. He found that the following hyperbolic form best fitted published data on weed competition:

$$D(W) = a/[1 + a/(bW)] \quad (2)$$

The parameter a can be interpreted as the asymptotic yield loss as $W \rightarrow \infty$. Crops typically give some positive yield even at very high weed densities, so a is normally less than one. The parameter b is the yield loss per weed as $W \rightarrow 0$.

W is a function of pre-treatment weed density (W_0) and $K(H)$, the proportion of weeds killed at herbicide rate H :

$$W = W_0 [1 - K(H)] \quad (3)$$

The kill function must be bounded by zero and one. It is usually represented in the literature by the following exponential function (e.g. Feder, 1979; Doyle *et al.*, 1984; Moffitt *et al.*, 1984; Auld *et al.*, 1987):

$$K(H) = 1 - \exp(-kH) \quad (4)$$

Substituting (2), (3) and (4) into (1) gives yield response as a function of herbicide dosage.

Profit (π) is given by:

$$\pi = P_y Y - P_h H - A - F \quad (5)$$

where P_y is output price, P_h is herbicide cost, A is herbicide application cost (which is independent of the application rate, H) and F represents costs from all other inputs which are assumed to be fixed at optimal levels.

Theoretical Analyses

In the following analyses, two decision frameworks are used. Firstly, the herbicide rate which maximises expected profit is derived. Although it selects economically efficient control strategies, this approach is not widely used in the weed economics literature. The usual approach is to fix the herbicide rate at some fixed 'recommended' rate and to calculate the threshold weed density above which it is worth applying that rate. The threshold density approach is the second framework used here.

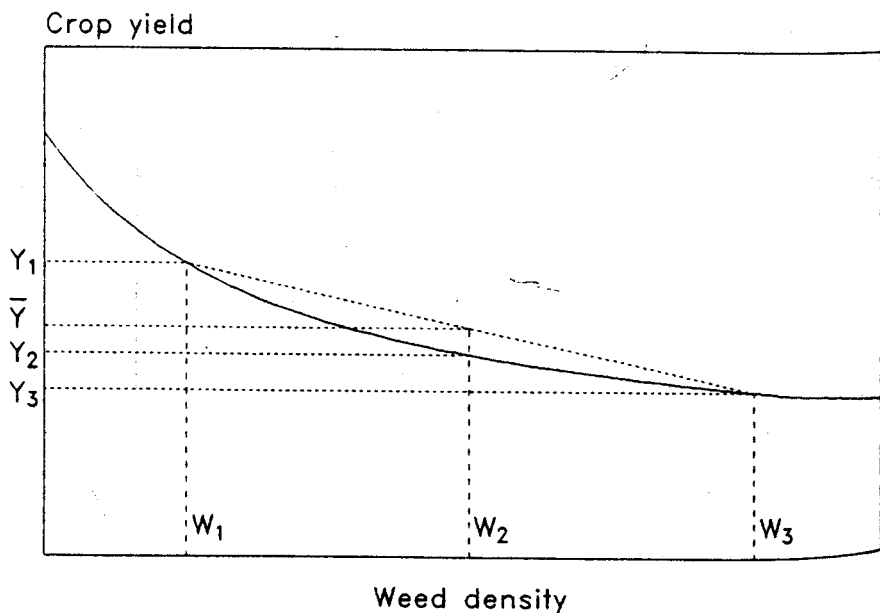
Stochastic Pre-Treatment Weed Density

There are two sources of stochasticity related to the initial weed density (W_0). One is uncertainty about the mean value of W_0 within the crop. The other is spatial variability in weed density on the ground. Weeds do not grow in

uniformly spaced formation but tend to grow in 'clumps' of high density mixed with areas of relatively low density. Greater variability of W_0 leads to less precision in inferences about the mean value of W_0 . The variability also means that, under the usual practice of applying a uniform herbicide rate to a large area of crop, some regions will receive less than their optimal dosage and others more. Auld and Tisdell (1986, 1987, 1988) have noted that uncertainty about, or variability in, W_0 can affect expected profit. They suggested that this would increase the threshold density, although they did not rigorously prove this claim. They also did not consider the question in the context of allowing herbicide rates to vary to suit economic and environmental conditions.

Figure 1 illustrates the way in which a stochastic initial weed density affects expected yield and, consequently, expected profit. Consider a crop containing a uniform distribution of weeds at density W_2 . The crop yield function shows that the crop would give a yield of Y_2 . Now consider a similar crop in which half the area of crop is infested with weeds uniformly distributed at density W_1 while the other half is infested at density W_3 . The mean weed density in this mixed crop is W_2 but the mean yield is \bar{Y} the average of Y_1 and Y_3 . Because of the convexity of the yield function, \bar{Y} is greater than Y_2 . In other words, the same number of weeds causes less yield loss if they are unevenly distributed than if they are evenly distributed. The biological explanation for this is that in the uneven distribution those weeds clumped together at high densities are forced to compete with each other for resources as well as with the crop plants. In these weeds, the competitive ability per weed plant is reduced so that, on average, yield loss is reduced.

Figure 1 Effect of Stochastic Weed Density on Expected Crop Yield



Now consider what effect this reduction in yield loss has on the optimal herbicide rate. It is shown in the Appendix that, as the variance of initial weed density increases, the herbicide rate which maximises expected profit decreases. This is counter to the usually presumed result that risk increases herbicide use. At least for decision makers whose objective is to maximise expected profits, uncertainty about W_0 reduces H^* . It does so because increasing the variance of weed density reduces the expected yield loss at a given mean weed density (after treatment), reducing the marginal productivity of herbicide.

Now consider the effect of σ_0^2 on the economic threshold, W_0^T , defined here as the lowest initial weed density at which the benefits of applying a fixed recommended herbicide dosage would exceed the costs. The derivation of the economic threshold is quite different to the derivation of the optimal dosage. The latter requires marginal analysis, as demonstrated in the Appendix, whereas the threshold is determined by comparing profits from two discrete input levels. The result of the marginal analysis is an input level, whereas the result of the threshold analysis is a pest density at which a fixed input level should be used. The threshold is derived as follows.

If the recommended herbicide rate (H_r) is used, profit is given by:

$$\pi(H_r) = P_y Y_0 [1 - D(W_r)] - P_h H_r - A - F \quad (6)$$

where W_r is weed density surviving application of the recommended herbicide rate. If no herbicide is applied, profit is:

$$\pi(0) = P_y Y_0 [1 - D(W_0)] - F \quad (7)$$

Without herbicide application, weed density is higher ($W_0 > W_r$), so the level of yield loss is greater. On the other hand, savings are made on herbicide costs ($P_h H_r$) and application costs (A). The threshold (W_0^T) is the lowest density at which application of H_r is at least as profitable as application of no herbicide, i.e. where $\pi(H_r) = \pi(0)$. Setting (6) equal to (7) and simplifying gives:

$$D(W_0^T) = \frac{P_h H_r + A}{P_y Y_0} + D(W_r) \quad (8)$$

This expression for the threshold applies to a deterministic model but here W_0 is a random variable, so that the interpretation of W_0^T is problematic. The most reasonable solution seems to be to re-define the threshold in terms of the mean weed density, \bar{W}_0 . Thus:

$$D(\bar{W}_0^T) = \frac{P_h H_r + A}{P_y Y_0} + D(\bar{W}_r) \quad (9)$$

If $\sigma_0^2 = 0$, equations (8) and (9) are equivalent, and $\bar{W}_0^T = W_0^T$. However, as we have seen in the previous discussion, if σ_0^2 is increased to a value greater than zero, the level of yield loss at a given mean weed density will be reduced. That is, both $D(W_0)$ and $D(\bar{W}_r)$ will be reduced by the increase in σ_0^2 . However, application of high rates of herbicide substantially reduces the variance of weed

density, so $\text{var}(W_e)$ will be very much less than $\text{var}(W_o)$. This also means that any change in $\text{var}(W_e)$ following an increase in σ_o^2 is small. Thus the main effect of an increase in σ_o^2 is a reduction in $D(W_o)$. But if the left-hand side of (9) is reduced while the right-hand side is almost unchanged, the equality will not be satisfied. To maintain the equality, it is necessary to increase the left-hand side by increasing W_o . W_o^T is the new, higher value of W_o which reinstates the equality. Thus, if the decision maker maximises expected profit, the economic threshold is increased by uncertainty about w_o . This is consistent with the findings above for H^* in that a higher threshold results in a lower expected level of herbicide use. Again, it contradicts the conventional wisdom in the literature that risk increases use of inputs for damage control.

Stochastic Weed Competitiveness

The actual level of yield loss per weed is unknown until the end of the growing season. In this analysis, uncertainty about weed competitiveness is characterised by uncertainty about the value of parameter b in the damage function (2). All other parameters and variables are assumed to be deterministic.

It is shown in the Appendix that an increase in the variance of b reduces the optimal herbicide rate. The result is similar to that for uncertainty about initial weed density, and it again runs counter to conventional wisdom about the effect of risk on herbicide use.

Uncertainty about b also affects the economic threshold. From equation (A18) in the Appendix, it can be seen that an increase in σ_b^2 decreases yield loss (γ is negative). The subsequent reduction in yield loss associated with untreated weeds will be much greater than the reduction in yield loss for the few weeds surviving treatment with the recommended dosage. In that case, an increase in the variance of b causes a larger decrease in the left-hand side of the threshold equation (8) than in the right-hand side. To maintain the equality, it is necessary to increase the left-hand side by increasing W_o . W_o^T is the new, higher value of W_o which reinstates the equality.

Again these conclusions are counter to the usual claims made about risk and herbicide/pesticide use. Uncertainty about weed competitiveness is another possible cause of reductions in herbicide use.

Stochastic Weed Kill

As well as uncertainty about the damage caused at a particular weed density, there can also be uncertainty about the number of weeds killed by a particular herbicide dosage. This seems to loom large in the minds of Western Australian wheat farmers. However, its effects on H^* and W_o^T are ambiguous. A direct effect of introducing stochasticity to the kill function is to reduce the level of weed mortality (in a similar fashion to that illustrated in Figure 1). This tends to increase the optimal herbicide rate. There is, however, a further impact of stochastic weed kill; it leads to uncertainty about W and, from the first part of this discussion, uncertainty about W reduces H^* . Thus uncertainty about weed kill has two effects on H^* : (a) expected weed survival is increased, which tends to increase H^* , and (b) weed density is made uncertain, decreasing H^* . The net effect depends on the balance of forces; numerical examples are presented later.

The total effect of stochastic kill on the economic threshold is also ambiguous. The direct effect of uncertainty about kill is a tendency to reduce

W_0^T , but stochastic kill also leads to uncertainty about the weed density which tends to increase W_0^T . The net effect depends on the convexity properties of the kill and damage functions.

Stochastic Herbicide Rate

Although the farmer selects the herbicide dosage to apply, he or she does not have perfect control over dosage. In particular, there is likely to be spatial variation in the dosage, especially with some of the modern herbicides which are applied at rates of just a few grams of active ingredient per hectare. Although the farmer controls the mean dosage applied to a crop, dosage received by different areas of the crop will follow some distribution. Chiao and Gillingham (1989) analysed the implications for optimal fertiliser practices of spatial variation in fertiliser rate.

The effects of stochastic herbicide rate on H^* and W_0^T are similar to the stochastic weed kill example above. Again there are two responses with an ambiguous net effect. Numerical examples are presented below.

Empirical Results

Method

The problem selected for empirical analysis was control of ryegrass in wheat by application of Hoegrass[®] (active ingredient diclofop-methyl). This problem was selected because of its economic importance in Western Australia where farmers consider ryegrass to be one of their most important crop weeds (Roberts *et al.*, 1988). The basic biological relationships were taken from Pannell (1990). Weed survival is given by:

$$W = W_0 / [1 + \exp(F)] \quad (10)$$

where

$$F = -2.85 - 0.995 \ln(H) - 0.00559 W_0 - 0.00366 \ln(H)W_0 \quad (11)$$

and W_0 is initial weed density. This function differs from the kill function in equation (4) in that the functional form is logistic and the proportion of weeds killed at a given herbicide dose is not independent of the weed density. The simplified version in (4) was adopted to facilitate theoretical analysis. Pannell (1990) estimated the following yield function:

$$Y = Y_0 (1 - 0.149H) \left[1 - \frac{0.544}{1 + 0.544/(bW)} \right] \quad (12)$$

where

$$b = 0.0172 \cdot \exp(-0.801Y_0) \cdot \exp(-5.70H) \quad (13)$$

where H is herbicide dose and Y_0 is weed-free yield. This function differs from the damage function given in equation (2) in two respects. Firstly, the parameter b (representing marginal yield loss per weed) is not fixed but depends on the weed-free yield and herbicide rate and, secondly, there is an additional term representing direct damage to the crop by herbicide.

Templates for a microcomputer spreadsheet program were developed for deriving optimal herbicide rates and density thresholds under uncertainty. The empirical analyses were conducted using mean values for costs, prices, weed densities and yields considered reasonable for the shire of Merredin in Western Australia's eastern wheatbelt: wheat price \$144/tonne, Hoegrass cost \$48 per kg a.i., weed-free yield 1.14 tonnes/ha, initial weed density 200/m² and recommended herbicide rate 0.375 kg active ingredient/ha.

Risk was included in the spreadsheets in two ways. In the case of the weed-free yield, the probability distribution was estimated by solving a biological simulation model of wheat growth for each of 76 years for which the required rainfall data were available for Merredin (1912 to 1987). The discrete distribution of 76 points obtained by this procedure was used directly in the spreadsheet. Each of the 76 values of weed-free yield were used to calculate actual yield for a particular set of assumptions regarding herbicide rate, weed density, etc. These were used to calculate the distribution of profit for given prices and costs. Finally, expected profit was calculated based on the assumption that each of the 76 observations was equally likely to occur. The distribution of weed-free yields obtained from the simulation model had a mean of 1.14 tonne/ha and a standard error of 0.626 tonne/ha.

Uncertainty about variables other than weed-free yield was represented in a similar way except that the probability distributions were generated by a normal random-number generator. For consistency with the yield data, each randomly generated distribution consisted of 76 observations. The coefficient of variation of weed density was estimated from field trials as 40 per cent. Coefficients of variation used for other variables were: herbicide rate, weed kill, and weed competitiveness, 40 per cent; output price, 20 per cent.

In the spreadsheet used to calculate optimal herbicide rates, the effect of risk on decision making was evaluated by directly calculating the solution which maximised expected profit. The herbicide rate was increased by discrete increments until expected profit started to decline. This was repeated with progressively smaller increments until an arbitrarily accurate solution was obtained. Checks were included to ensure that the solution obtained was a global optimum.

A somewhat similar approach was taken in the threshold spreadsheets except that the variable being solved for was the weed density and the criterion for stopping was when expected profits from treatment with the recommended dose just exceeded expected profits from non-treatment. In both spreadsheets, solutions were obtained using a combination of spreadsheet formulae to calculate expected profit for a given set of parameter values and macro programs to control changes in parameter values and tests for optimality/thresholds.

Results

Results are shown in Table 1. First, consider the results for the variables considered individually in the earlier theoretical analyses. Results for W_0 and b are consistent with the theoretical findings that uncertainty decreases H^* and

increases W_0^T . However, despite this consistency with the theory, it can be seen that the magnitudes of the effects are very small. At least for this problem and this set of assumptions, it seems that the effect of uncertainty about W_0 or b on expected profit is unlikely to significantly affect farmer behaviour.

Theoretical results for the effects on H^* of uncertainty about weed kill and herbicide rate were ambiguous, but in each empirical example H^* is reduced by uncertainty. At the same time density thresholds are increased. However, as with H_0 and b , the effects are quite small.

Table 1 Optimal Herbicide Rates (H^*) and Weed Density Thresholds (W_0^T) for Risk-Neutral Decision Makers Under Different Sources of Uncertainty

Stochastic Variable	H^* (kg a.i./ha)	W_0^T (plants/m ²)
None	0.265	40
Initial Weed Density (W_0)	0.264	43
Weed Competitiveness (b)	0.263	43
Weed Kill (K)	0.262	41
Herbicide rate (H)	0.259	43
Weed-Free Yield (Y_0)	0.249	51
Y_0 and W_0	0.248	53
Y_0 and H	0.237	57
Y_0 , H and W_0	0.241	58

Table 1 also includes results for weed-free yield. This was not included in the theoretical analysis because the simplified response model did not include the mechanism by which uncertainty about Y_0 affects expected profit. In the more detailed model of equations (10) to (13), Y_0 affects weed competitiveness but it does so non-linearly. The result is that uncertainty about Y_0 increases expected yield loss, reducing expected profit. In the empirical model, uncertainty about Y_0 , like all the other variables, reduced H^* and increased W_0^T . The impact of Y_0 on H^* and W_0^T was greater than any of the other individual variables.

The final three sets of results in Table 1 show that, even if individual sources of risk have a small impact on decision making, in the realistic case of multiple sources of risk the effects can be more substantial. If the effect of multiple sources of risk on expected profit are considered, H^* can be reduced by over 11% and W_0^T increased by as much as 45%.

Conclusion

In all the numerical results presented here, the effect of risk on a risk-neutral decision maker was to reduce herbicide use, either by reducing the optimal herbicide rate or by increasing the weed threshold for herbicide use. This runs directly counter to the usual presumption about the impact of risk on use of herbicides and other types of pesticides under risk aversion. Whether the effect of risk aversion on herbicide use is sufficient to counter these effects is a subject for further investigation.

The reason for the decline in herbicide use is that risk reduces the marginal productivity of herbicide. There are two ways in which this can occur.

Uncertainty related to weed mortality increases the mean weed density surviving treatment and at higher mean weed densities, the marginal yield loss per weed is lower. Uncertainty related to weed competition reduces the mean yield loss per weed directly. In both cases, the effect is due to the convexity of the relevant biological relationships.

Results of the study highlight the importance of using biologically realistic relationships for analysis of response. If linear approximations had been used, the implications of risk for decision making under expected profit-maximisation would not have been apparent.

Another general conclusion to be drawn is that it can be important to consider multiple sources of risk in estimating the magnitudes of effects of risk on decision making. Although individual sources of risk had small effects on optimal decision making for weed control, combinations of risky variables had sizeable impacts.

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APPENDIX

Effect of Stochastic Weed Density on Optimal Herbicide Dose

The aim of the following analysis is to determine whether greater variance of initial weed density (W_0) increases or reduces the optimal herbicide rate (H^*), i.e. to determine the sign of $\partial H^* / \partial \text{var}(W_0)$. Assume that a farmer has a subjective distribution of initial weed density such that $W_0 = \bar{W}_0 + \epsilon_0$ where \bar{W}_0 is the mean of the distribution and ϵ_0 is a random variable with mean zero. After application of herbicide, a number of weeds are killed so the weed density is reduced from W_0 to W . The variance in W_0 leads to variance in W , i.e. $W = \bar{W} + \epsilon_w$ where ϵ_w is another random variable with mean zero. Note that ϵ_w is simply a transformation of ϵ_0 . In this analysis it does not incorporate uncertainty about the level of weed kill. Although herbicide application reduces the variance of W to less than that of W_0 it is not reduced to zero. For convenience, it will be assumed that post-treatment weed density is normally distributed, i.e. $\epsilon_w \sim N(0, \sigma_w^2)$. The distribution of weed density may not be normal in practice (indeed it cannot be since it must be truncated at zero), but examination of weed count data from field trials in Western Australia shows that the normal distribution gives a reasonable approximation of the actual distribution. From (1) and (5), profit (π) is given by:

$$\pi = P_y Y_0 [1 - D(W)] - P_h H - A - F \quad (A1)$$

An approximation will simplify the analysis: the damage function, $D(W)$, is approximated by a second-order Taylor series approximation about \bar{W} , the expected value of W .

$$D(W) \approx D(\bar{W}) + (W - \bar{W}) D'(\bar{W}) + 0.5 (W - \bar{W})^2 D''(\bar{W}) \quad (A2)$$

where primes denote derivatives. This can be rearranged to give:

$$D(W) \approx \alpha + \beta W + \gamma W^2 \quad (A3)$$

where $\alpha = D(\bar{W}) - \bar{W} D'(\bar{W}) + \frac{1}{2} \bar{W}^2 D''(\bar{W})$

$$\beta = D'(\bar{W}) - \bar{W} D''(\bar{W})$$

$$\gamma = \frac{1}{2} D''(\bar{W})$$

Note that $\beta > 0$ since:

$$D'(W) = \frac{a^2}{bW^2(1 + a/(bW))^2} > 0$$

and

$$D''(W) = D'(W) \frac{a^2}{b} \left[\frac{-2}{W} - \frac{a}{bW^2(1 + a/(bW))} \right] < 0$$

Also note that this latter result implies that γ is negative. The use of this approximation is defended on the grounds that the important features of the damage function for this analysis are preserved, in particular that $D'(W) > 0$ and $D''(W) < 0$. Now substituting (A3) into the profit function (A1) gives:

$$\pi \approx P_y Y_0 [1 - \alpha - \beta W - \gamma W^2] - P_h H - A - F \quad (A4)$$

The objective is to maximise expected profits. Since it is assumed that weed density is the only uncertain variable and that it is normally distributed, expected profit is given by:

$$E(\pi) \approx P_y Y_o \left[1 - \alpha - \beta \bar{W} - \gamma (\bar{W}^2 + \sigma_w^2) \right] - P_h H - A - F \quad (A5)$$

The first- and second-order conditions for selecting the herbicide rate which maximises expected profit are:

$$\partial E(\pi) / \partial H = 0 \quad (A6)$$

and

$$\partial^2 E(\pi) / \partial H^2 < 0 \quad (A7)$$

The response model presented above can have regions where (A7) is not satisfied as well as regions where it is. We will assume here and in the subsequent discussion that the optimal herbicide rate is correctly determined in a region where (A7) is satisfied. This means that (A6) becomes a necessary and sufficient condition for expected profit maximisation. From (A5) the first-order condition is:

$$\partial E(\pi) / \partial H = P_y Y_o \left[-\beta \partial \bar{W} / \partial H - \gamma (2\bar{W} \partial \bar{W} / \partial H + \partial \sigma_w^2 / \partial H) \right] - P_h = 0 \quad (A8)$$

Although there is no closed form solution for H^* available from (A8), Hey (1981, p.38) demonstrates how the comparative-static properties of a solution like this can be found by total differentiation:

$$\frac{\partial H^*}{\partial X} = - \frac{\partial^2 E(\pi)}{\partial H \partial X} \quad \frac{\partial^2 E(\pi)}{\partial H^2} \quad (A9)$$

where X is a variable in (A8) and the partial derivatives on the right-hand side of (A9) are evaluated at H^* . But to ensure that H^* is a maximum, we have already assumed in (A7) that $\partial^2 E(\pi) / \partial H^2$ is negative. Therefore the sign of $\partial H^* / \partial X$ depends on the sign of the cross partial derivative, $\partial^2 E(\pi) / \partial H \partial X$, evaluated at H^* .

$$\frac{\partial H^*}{\partial X} \begin{matrix} \geq \\ \leq \end{matrix} \text{ as } \frac{\partial^2 E(\pi)}{\partial H \partial X} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (A10)$$

This result will be used in this and the next section.

Return now to the problem of finding the direction of response to an increase in $\text{var}(W_o)$ (i.e. the sign of $\partial H^* / \partial \sigma_o^2$). From (A10) we seek the sign of $\partial^2 E(\pi) / \partial H \partial \sigma_o^2$ and from (A8),

$$\partial^2 E(\pi) / \partial H \partial \sigma_o^2 = -P_y Y_o \gamma \cdot \partial^2 \sigma_w^2 / \partial H \partial \sigma_o^2 \quad (A11)$$

To clarify what this means, $\partial \sigma_w^2 / \partial H$ is the rate at which the variance of weed density changes in response to changes in herbicide rate. Examination of field trials shows that variance of weed density decreases with increased herbicide rate. The issue here is whether σ_w^2 decreases at a higher or lower absolute rate if σ_o^2 is increased; i.e. is $\partial^2 \sigma_w^2 / \partial H \partial \sigma_o^2$ positive or negative. In addressing this, recall that $W = W_o (1 - K)$. Assuming that W_o is normally distributed [$W_o \sim N(\bar{W}_o, \sigma_o^2)$],

$$\sigma_w^2 = \text{var}(W) = \text{var}[W_o(1 - K)] = \sigma_o^2(1 - K)^2 \quad (A12)$$

$$\partial \sigma_w^2 / \partial H = \sigma_o^2(2)(1 - K)(-\partial K / \partial H) \quad (A13)$$

Now, taking the partial derivative with respect to σ_0^2 ,

$$\partial^2 \sigma_w^2 / \partial H \partial \sigma_0^2 = (2) (1 - K) (-\partial K / \partial H) \quad (\text{A14})$$

which, when substituted into (A11), gives:

$$\partial^2 E(\pi) / \partial H \partial \sigma_0^2 = P_y Y_o \gamma (2) (1 - K) (\partial K / \partial H) \quad (\text{A15})$$

which is negative since γ is the only negative term. \bar{S}_0 , from (A10), $\partial H^* / \partial \sigma_0^2$ is also negative.

Effect of Stochastic Weed Competitiveness on Optimal Herbicide Dose

Consider the actual yield function:

$$Y = Y_o [1 - \Phi(W)] = Y_o \left[1 - \frac{a}{1 + a/(bW)} \right] \quad (\text{A16})$$

where Y_o is weed-free yield, a is the asymptotic yield loss as $W \rightarrow \infty$ and b is the yield loss per weed as $W \rightarrow 0$. The damage function can be approximated by a second-order Taylor series as follows:

$$D \approx \alpha + \beta (bW) + \gamma (bW)^2 \quad (\text{A17})$$

where $\beta > 0$ and $\gamma < 0$. Note that α , β and γ are different in this example than in the previous example. Now let the parameter b be a random variable $b = \bar{b} + \xi$ where $\xi \sim N(0, \sigma_b^2)$. The problem is to find the direction of response to an increase in $\text{var}(b)$ (i.e. the sign of $\partial H^* / \partial \sigma_b^2$). From (A10) we seek the sign of $\partial^2 E(\pi) / \partial H \partial \sigma_b^2$. Expected damage is:

$$E(D) \approx \alpha + \beta \bar{b} W + \gamma W^2 (\bar{b}^2 + \sigma_b^2) \quad (\text{A18})$$

and expected profit is:

$$E(\pi) \approx P_y Y_o \left[1 - \alpha - \beta \bar{b} W - \gamma W^2 (\bar{b}^2 + \sigma_b^2) \right] - P_h H - A - F \quad (\text{A19})$$

It follows that:

$$\partial E(\pi) / \partial H = P_y Y_o \left[-\beta \bar{b} \partial W / \partial H - \gamma 2W \partial W / \partial H (\bar{b}^2 + \sigma_b^2) \right] - P_h = 0 \quad (\text{A20})$$

and

$$\partial^2 E(\pi) / \partial H \partial \sigma_b^2 = -2 P_y Y_o \gamma W \partial W / \partial H \quad (\text{A21})$$

which is negative. Given equation (A21), equation (A10) implies that an increase in $\text{var}(b)$ reduces the optimal herbicide rate.