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Biological Interactions and Environmental Effects in the Economics of Pest Control[1, 2]

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The problem of pest control is tackled in a context of an ecosystem that consists of prey–predator populations with human interaction through pesticide application. The control, aimed at reducing pest damage, results in two undesirable external effects: reduction of beneficial predator population and environmental contamination. The untapped natural equilibrium is compared with equilibrium resulting from decentralized and centralized economic decision making. It is shown that, under certain conditions, myopic decision rules increase rather than decrease the pest damage. The "user cost" (or benefit) is shown to be crucial in determining the optimal centralized policy, and its relations to the various components of the system are analyzed. The components of the user cost are analyzed to determine the level of taxes or subsidies that will yield the optimal policy.

1. INTRODUCTION

"With the slow development of civilisation, so has man gradually realized the extent to which pests harm his crops, annoy him and transmit diseases, both human and those of domestic animals" [6, p. 1]. As rapid world population growth emphasizes the need for increased supplies of food and other agricultural resources, the problem is aggravated; and improvement of pest management may become crucial. Pest-borne diseases, which have been almost completely controlled in the developed world, are still a grave problem in the underdeveloped parts of the world. It has been suggested that 10–20% of world crops and stocks are destroyed each year by pests although extensive pest control methods are maintained [2, p. 29]. Edwards [6] reports that pest damage in the United States is of the order of $4 billion, while the figure for the world as a whole is $21 billion. Knipling [14] has estimated that, in the first decade of the use of DDT, 5 million lives have been saved and 100 million serious illnesses have

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been prevented. These figures demonstrate the importance of the need to control pests, a task currently performed mainly through chemical pesticides. Although pesticides have been used for many generations, the massive use of synthetic pesticides began about 1940. At the present time, approximately 1000 varieties of pesticides are in common use in the world; and the volume of their production in the United States alone rose from 0.3 billion pounds in 1954 to 1.2 billion pounds in 1973 [6]. The widespread applications of pesticides has focused attention on the harmful impact of pesticides, namely, the contamination of the environment by pesticide residues. There has been an everincreasing stream of reports on residue accumulation in the soil, water, air, and most parts of the food chain; these have aroused serious concern, as many of the residues are deleterious for man and animals.

This concern over the extensive use of pesticides should obviously focus attention on the various elements in the ecosystem that affect pest population (besides pesticides), e.g., predators, parasites, and intraspecies competition. Interspecies interaction in biological ecosystems has been discussed in the literature (e.g., [16, 19]). However, these elements have been ignored by most economists dealing with pest management [11–13, 27]. Shoemaker [23, 24], in a dynamic programming model, incorporated prey–predator interaction as well as pesticide residues and obtained a numerical solution to a specific example. However, her analysis is operation-research oriented and does not explore the general economic implications of the results.

In the present paper an economics-oriented, pest control model that incorporates both the environmental effect of pesticide residue and species interaction within the ecosystem is suggested. The analysis focuses on the economic implications of the decision-making procedures, emphasizing the common property features of pest problems. Although each decision maker may have a considerable degree of control over the pest population in his domain, population dynamics is determined by the total population of the region over which he has negligible control. The resulting externalities have been extensively discussed in the literature on common property resources [1, 4, 8, 22, 26]. However, there is a difference between the usual natural resource problems and the present one, since in the first case, a beneficial factor is being ignored, while in the latter case, there are both beneficial (predator) and detrimental (pest) factors. Another feature that distinguishes the current problem from the regular natural resource problems is the inability in most cases to avoid damage to the predator when pesticides are applied. Quirk and Smith [19] describe a case of prey–predator interaction in fishery; in their case each species is controlled separately, and both species are beneficial. Although directed to the problem of pest control, the analysis can be extended to interacting biological natural resources in general (fisheries, wildlife management, etc.). Furthermore, as noted by Samuelson [21], prey–predator relations could be interpreted in a broader sense, such as the relation between capitalists and laborers.

The residue effect presents an additional type of externality since individual decision makers (farmers and households) do not include this effect in their decisions.

The program of the paper is as follows. The biological and economical components of the system are presented, followed by an analysis of the outcome of decentralized decision making. After these results are compared with the "natural" situation (i.e., before pesticide application), the centralized decision problem and results are discussed, while comparisons between the centralized and decentralized cases are made. The last section summarizes the discussion and adds some policy implications.

2. BIOLOGICAL COMPONENTS

Consider an ecosystem composed of two biological populations. The first, referred to as pest or prey and denoted by N, affects the human well-being either through reduction of profits or by a direct negative effect on the utility function. The second population, hereafter called predator and denoted by P, reduces the growth rate of the pest either by feeding on it directly or by exploiting it in some other way (parasitism).[3] The predator is assumed to have no direct effect on production or utility. Both N and P denote total populations in the region under consideration.

The population dynamics (without human intervention) is described by[4]

$$(1/N)(dN/dt) \equiv \dot{N}/N = g[N(t), P(t)], \tag{1}$$

$$(1/P)(dP/dt) \equiv \dot{P}/P = f[N(t), P(t)]. \tag{2}$$

These functions are assumed to possess the properties

$$\text{(a) } \frac{\partial g}{\partial N} \equiv g_1 < 0, \quad \text{(b) } \frac{\partial g}{\partial P} \equiv g_2 < 0, \quad \text{(c) } \frac{\partial f}{\partial P} \equiv f_2 < 0, \quad \text{(d) } \frac{\partial f}{\partial N} \equiv f_1 > 0; \tag{3}$$

$$\text{(a) } g(0,0) > 0, \quad \text{(b) } (0, \alpha_1) = 0, \ \alpha_1 > 0, \quad \text{(c) } g(\alpha_2, 0) = 0, \ \alpha_2 > 0; \tag{4}$$

$$f(\alpha_3, 0) = 0, \quad 0 < \alpha_3 < \alpha_2. \tag{5}$$

Equations (1) and (2) describe the relative growth rates of the pest and its predator, respectively, where each is a function of both populations. The intraspecies relations are given by (3a) and (3c), which reflect the negative impact of increasing a species population on its own rate of growth due to competition [18, pp. 81–87]. The interspecies interactions, (3b) and (3d), characterize the prey–predator type of relations where growth of the prey is inhibited by increased predator population, while predator growth is positively affected by abundance of the prey population. Properties (4) and (5) [18, pp. 87, 88; 20, p. 378] describe the behavior of the populations at extreme levels. If both populations are very small, the pests multiply (4a). If there are too many predators, the pests cannot multiply (4b). There is a limit to the pest-carrying capacity of the environment even in the absence of predators (4c). If the prey is scarce, the predator cannot multiply (5). Equation (5) is a condition for nontrivial natural equilibrium since, if $\alpha, \leq \alpha_3$, the predator will disappear in a finite time, and an equilibrium will be maintained at $N = \alpha_2, P = 0$. The following proposition deals with an interior steady state (ss) in a natural environment.

PROPOSITION 1. *Under assumptions (1)–(5), there exists a unique interior ss* (N^0, P^0)*; and this ss is locally stable.*

Proof. The implicit function $g(N, P) = 0$,[5] is a curve in the (N, P) space with a slope $dP/dN = -g_1/g_2 < 0$ by (3); and the points $(0, \alpha_1)$ and $(\beta, 0)$ belong to this curve by (4b) and (4c).

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[3] A general treatment of prey–predator dynamics is given by Samuelson [21].

[4] The justification for considering only one pest and one predator is twofold: (1) In many pest problems only one major pest is involved, and each pest usually has one major predator or parasite that inhibits its growth; and (2) the analysis is greatly simplified but still preserves the general nature of prey–predator relations and economic implications.

[5] Whenever possible, the time variable t is hereafter omitted for notational simplification.

The function $f(N,P)=0$ is a curve in the (N,P) space with a slope $dP/dN = -f_1/f_2 > 0$ by (3); and the point $(\alpha_3, 0)$ belongs to that curve by (5). Since $\alpha_2 > \alpha_3$ by (5), the two curves must intersect; and there is only one intersection point. Denote this intersection point by (N^0, P^0). Local stability may be shown by linearizing systems (1) and (2) via the Taylor expansion around (N^0, P^0). The point (N^0, P^0) is locally stable if $g_1f_2 - f_1g_2 > 0$, and the trace $(Ng_1 + Pf_2) < 0$. Both conditions hold by (3). Q.E.D.

Note that local stability implies global stability if there exists a Lyapunov function [18, p. 15]. Such a function obviously exists when $g(\cdot)$ and $f(\cdot)$ are linear. Existence of stability in a broader sense (which also allows for limit cycles) requires the Kolmogorov conditions to hold [20]. These conditions are $Ng_1 + Pg_2 < 0$ and $Nf_1 + Pf_2 > 0$ in addition to our model assumptions (3b), (3c), (4a)–(4c), and (5). Observe that the only condition that may not be met is $Nf_1 + Pf_2 > 0$. However, there exists some subspace in the nonnegative quadrant of the (N,P) space for which this requirement holds. In the complement of this subspace, extinction of the predator is possible.

Figure 1 illustrates the behavior of the populations in the (N,P) space. The direction of the vertical and horizontal arrows may be verified directly by (3).

3. ECONOMIC COMPONENTS

The economic model is based on the following setup. Within a given region, which is closed to outside migration of pest and predator populations, there are numerous identical decision units (e.g., farms, households). Both pest and predator populations are equally distributed among decision units. Pests (and predators) are usually mobile and move from one decision unit to another. Biological populations differ in their degree of mobility; some species stay at a given location (decision unit) as long as a whole agricultural season (alfalfa weevil), while others may stay at one location for only one night (mosquitoes). It is natural, then, to view the problem in a discrete time

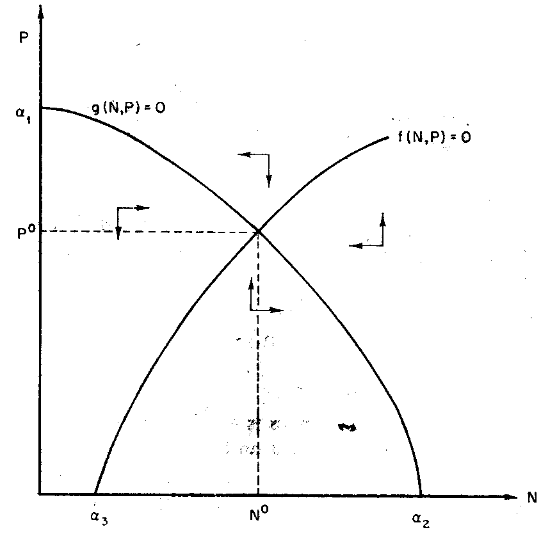

Figure 1

framework where the length of the periods is determined by the degree of pest mobility.[6]

While staying within a decision unit, pests cause damage that is obviously increasing with their number. It therefore follows that, by reducing the number of pests present at a given period in a given decision unit, the amount of damage inflicted would diminish. This explains why individual farmers, for example, are willing to apply pesticides and bear their costs. It is important to note, however, that because of their mobility, the dynamics of pest (and predator) populations within a decision unit depends upon the total population levels in the region as a whole rather than on the number present within a decision unit. Since the effect of a single decision maker's action on the total pest population is negligible, he views the level of pest population in the next period as an exogenous variable that is currently beyond his control. Nevertheless, the combined actions of all individual decision makers do affect pest dynamics.[7] As individual decision makers are aware of their inability to influence at any period future pest populations in their own domain, they have no incentive to consider periods other than the present one. If, on the other hand, the whole region were under supervision of one decision maker only, he would obviously have a long-run planning horizon.

The economic effects of the pest are (1) the direct loss to society through damaged crops and merchandise or through annoyance and health problems, and (2) the costs incurred in attempting to control the pest, including monetary cost and environmental effects.

Define x_t as a composite pest control input during period t per decision unit that may include pesticides, equipment, and labor combined in an "optimal" way. The control affects both pest and predator populations through dosage response functions, often referred to as kill functions. It is widely accepted [15, 24, 27] that the proportion of pests killed (k) is independent of the pest level and is a monotonically increasing function of the control (x). We assume that it is concave, reflecting diminishing marginal response:

$$k = k(x), \quad k_x > 0, \quad k_{xx} < 0 \lim_{x \to \infty} k(x) = 1, \tag{6}$$

where k is defined for a decision unit per time period.

The same reasoning applies for the dosage response of predators affected by the application of x, although this is not intended. Define h as the proportion of predators killed; then

$$h = h(x), \quad h_x > 0, \quad h_{xx} < 0 \lim_{x \to \infty} h(x) = 1. \tag{7}$$

The direct cost of x per decision unit per time period is defined by a monotonically increasing convex function

$$\tilde{C} = \tilde{C}(x), \quad \tilde{C}_x > 0, \tilde{C}_{xx} \geqq 0. \tag{8}$$

The accumulated stock of pesticide residue is denoted by R_t; its time rate of change

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[6] In the extreme case of continuous mobility, one may define the length of the time period as the time duration of control effect, which, in the case of certain pesticides, may last several days.

[7] This is analogous to the situation under perfect competition where the actions of individual agents do not affect the market price when taken separately, but their combined effect determines the price.

is measured in the same units as x and defined by

$$R_{t+1} - R_t = nx_t - \theta R_t, \tag{9}$$

where n is the number of (identical) decision units, assumed constant, and θ is a positive constant rate of degradation of the residue. An explanation and discussion of such functions appear in [17, p. 64]. Note that, for any initial level of residue R_0 and any fixed level of control ($\bar{x} \geq 0$), R_t converges to an ss level of the residue $\bar{R}(\bar{x}) = n\bar{x}/\theta$ with a speed depending upon $|R_0 - \bar{R}|$ and on θ.

The environmental costs of control are not easily measurable, but interest is focused here on the qualitative effect of these costs, for which it is sufficient to assume the cost function

$$E = E(R) \geq 0, \quad E_R \geq 0. \tag{10}$$

The net growth rates of the two populations are redefined to account for both the discrete time formulation and the effects of the control. Note first that, in the absence of control, (1) and (2) are substituted by $(N_{t+1} - N_t)/N_t = g(N_t, P_t)$ and $(P_{t+1} - P_t)/P_t = f(N_t, P_t)$, with (3)–(5) unchanged. It will be assumed for simplicity that control is applied at the beginning of any period t so that only the surviving pest can cause damage and reproduce. Since all decision units are identical (and thus are induced to apply the same level of control), the proportion killed in any one decision unit is the same for all units and is equal to the proportion killed in the region as a whole.[8]

Thus,

$$N_{t+1} = N_t[1 - k(x_t)] + N_t \cdot g\{N_t[1 - k(x_t)], P_t[1 - h(x_t)]\}[1 - k(x_t)], \tag{11}$$

which is rearranged as

$$\begin{aligned} \frac{\Delta N_t}{N_t} \equiv \frac{N_{t+1} - N_t}{N_t} &= g[N_t(1 - k), P_t(1 - h)](1 - k) - k \\ &\equiv \phi^1(N_t, P_t, x_t). \end{aligned} \tag{12}$$

Similarly, for the predator,

$$\begin{aligned} \frac{\Delta P_t}{P_t} \equiv \frac{P_{t+1} - P_t}{P_t} &= f[N_t(1 - k), P_t(1 - h)](1 - h) - h \\ &\equiv \phi^2(N_t, P_t, x_t), \end{aligned} \tag{13}$$

where x_t is deleted from $k(x_t)$ and $h(x_t)$ for notational simplification.

Consider the effect of an increase in the level of x_t on the pest population at the beginning of the following period (N_{t+1}). On the one hand, the rate of growth (g) will increase when compared with the rate that would have prevailed with the lower level of x_t (because of reduced intraspecies competition and the decline in predation). On the other hand, the starting population level (after application) is lower with higher x_t. Thus, the net result on N_{t+1} seems to be undetermined. However, it has been observed

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[8] The assumption of identical decision units is not restrictive since one may define all variables per some standard unit of measure (say, acre); and then, provided that there are no economies of scale, all the results hold.

that, in most cases, the time of immobility is relatively short so that pest population in the following period (N_{t+1}) decreases as pesticide application (x_t) increases.[9] In the case of the predator, the change in P_{t+1}, as a result of a change in x_t, is definitely negative since, if prey were to remain in the preapplication level, P_{t+1} is necessarily lower for higher x_t. The effect of reducing prey population further decreases $f(\cdot)$ and thus, P_{t+1}. The implications of this discussion are summarized in the following.

$$\begin{aligned} \frac{1}{N_t}\frac{\partial N_{t+1}}{\partial x_t} &\equiv -G = -k_x(g+1) - (1-k)(g_1Nk_x + g_2Ph_x) < 0, \\ \frac{1}{P_t}\frac{\partial P_{t+1}}{\partial x_t} &\equiv -F = -h_x(f+1) - (1-h)(f_1Nk_x + f_2Ph_x) < 0. \end{aligned} \tag{14}$$

It should be noted (as will become apparent later) that (14) does not imply that pest population is necessarily reduced in the long run by pesticide application, compared with a "no pesticide" (natural) situation.

Damage can generally be described as an increasing convex function of pests remaining after applying the control x at the beginning of the period. Some writers dealing with pest management have assumed a linear damage function (e.g., Hall and Norgaard [11], Talpaz and Borosh [27], Shoemaker [24]). Results are not affected by this simplifying assumption; thus, it has been assumed heretofore (without loss of generality). The damage function for a single decision unit during one time period is thus,

$$D\{(N_t/n)[1-k(x_t)]\} = \delta\cdot(N_t/n)\cdot[1-k(x_t)], \quad \delta > 0. \tag{15}$$

4. DECENTRALIZED DECISION MAKING

As discussed before, each decision unit has a relatively small portion of total pest population in its area.[10] Changes in pest population within one unit, therefore, have only a negligible effect on the total population. However, the portion present in a field at a given point of time determines the instantaneous damage to that unit. Therefore, there is an incentive for a single decision maker to apply pesticides and thereby reduce the damage in a single time period; however, he will ignore the effects on the dynamics of the total pest population, which is beyond his control. Similar arguments hold for predator population, which may also be ignored because of lack of information. Pest and predator populations, therefore, should be regarded as common property resources. This type of externality has been termed by Smith [26] as "stock externality." The environmental effects of pesticide residue are the "classical" type of production externalities. Since no market prices reflect these effects, decision makers do not consider them. As a result, each decision maker behaves myopically and minimizes the current period's loss:[11]

$$\min_{x_t} L(N_t, x_t) = \delta(N_t/n)[1-k(x_t)] + \tilde{C}(x_t), \quad x_t \geq 0. \tag{16}$$

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[9] Gutierrez *et al.* [10], Ehler *et al.* [7] and Gutierrez [9].

[10] This is not always the case, as there are instances of "sole ownership," i.e., isolated farms, a single grain mill, etc. The analysis of the next section applies also to such cases.

[11] In the case of utility loss, the decision maker will seek to maximize $U[(1-k)N, M]$, where U is a utility function with $(1-k)N$ remaining pest population, and M is income net of control costs.

The necessary condition for optimum is

$$L_x = -\delta(N/n)k_x + \tilde{C}_x \geqq 0, \quad xL_x = 0. \tag{17}$$

This condition is also sufficient by the concavity of $k(x)$ and convexity of $\tilde{C}(x)$.

Equation (17) implies as usual the equality of marginal cost to marginal benefit. The marginal benefit is the damage avoided by the marginal unit of control. For $x > 0$, the decision rule implied by (17) is

$$x = \psi(N), \tag{18}$$

with

$$\begin{aligned}\psi_N &= \delta k_x/(n\tilde{C}_{xx} - \delta N k_{xx}) \\ &= \delta k_x/(C_{xx} - \delta N k_{xx}) > 0,\end{aligned} \tag{19}$$

where $C(x) = n\tilde{C}(x)$, the total pesticide cost for the society.

If an interior ss exists,[12] then, by (12), (13), and (18), it must satisfy

$$\phi^i[N, P, \psi(N)] = 0, \quad i = 1, 2, N > 0, P > 0. \tag{20}$$

Local stability of this system requires that the trace $N\phi_N^1 + P\phi_P^2 < 0$ and the determinant $\phi_N^1\phi_P^2 - \phi_N^2\phi_P^1 > 0$, where the derivatives are evaluated at the ss point. The trace condition holds, since by (3), (12), (13), (14), and (19),

$$N\psi_N^1 + P\psi_P^2 = N[(1-k)^2 g_1 - \psi_N G] + (1-h)^2 P f_2 < 0.$$

However, the sign of the determinant is not known a priori. By (12), (13), and (14),

$$\begin{aligned}\phi_N^1\phi_P^2 - \phi_N^2\phi_P^1 &= (1-k)^2(1-h)^2(g_1 f_2 - g_2 f_1) \\ &\quad - \psi_N \cdot (1-h)[(1-h)f_2 G - (1-k)g,F] = (v-z)\psi_N \cdot (1-h),\end{aligned}$$

where

$$z(N, P, x) = (1-h)f_2 G - (1-k)g_2 F \tag{21}$$

and

$$v(N, P, x) = (g_1 f_2 - g_2 f_1)(1-k)^2(1-h)/\psi_N, \tag{22}$$

by (3) and (19). Thus, local stability requires that $z < v$, evaluated at the ss point. If $z > v$, this ss is a saddle point, and multiple equilibria are possible.

The following proposition relates a ss point of system (20) to the ss considered in Proposition 1 (the natural ss).

PROPOSITION 2. *Given a natural interior ss (N^0, P^0) satisfying $g(N, P) = f(N, P) = 0$, an interior ss (N^1, P^1) $[\neq(N^0, P^0)]$, defined by (20) (if it exists), must satisfy*

(*a*) $P^1 < P^0$,

(*b*)
$$\begin{array}{lll} & > N^0 & \qquad\qquad > Z \\ N^1 & = N^0 \quad \textit{if } z(P, N, \psi) & = 0 \quad \textit{for } N \in [N^1, N^0], P \in [P^1, P^0]. \\ & < N^0 & \qquad\qquad < 0 \end{array}$$

Proof. Consider the system defined by (12) and (13) with

$$\phi^i(N, P, x) = 0, \quad i = 1, 2. \tag{23}$$

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[12] By an interior ss, we mean that $N > 0, P > 0$, and $x > 0$. By (10), $x > 0$ implies that at ss $R > 0$. Since there is a one-to-one relation between an ss x and an ss R, the latter is omitted from the notation of the ss point.

For any given $x \geq 0$, the solution of (23) defines a point of ss; and the set of solutions (for various x levels) is the locus of ss points. Obviously, (N^0, P^0), (N^1, P^1) belong to this locus. Differentiate (23) to obtain

$$\left.\frac{dP}{dx}\right|_{\phi^1=\phi^2=0} = \frac{(1-k)^2 g_1 F - (1-k)(1-h) f_1 G}{(g_1 f_2 - g_2 f_2)(1-k)^2(1-h)^2} < 0; \tag{24}$$

by (3), (14), and (19),

$$\begin{aligned}\left.\frac{dN}{dx}\right|_{\phi^1=\phi^2=0} &= \frac{(1-h)^2 f_2 G - (1-k)(1-h) g_2 F}{(g_1 f_2 - g_2 f_1)(1-k)^2(1-h)^2} \\ &= z/(g_1 f_2 - g_2 f_1)(1-h)(1-k)^2 \gtreqless 0 \end{aligned} \tag{25}$$

if and only if $z \gtreqless 0$, where z is as defined in (21). Since

$$\phi^1(N^0, P^0, 0) = \phi^2(N^0, P^0, 0) = 0,$$
$$\phi^1[N^1, P^1, \psi(N^1)] = \phi^2[N^1, P^1, \psi(N^1)] = 0,$$

and

$$\psi(N^1) = x^1 > 0,$$

it follows by (24) that $P^1 < P^0$. If the sign of z does not change for $N \in [N^0, N^1]$, $P \in [P^0, P^1]$, the signs of the derivative established in (25) hold for finite changes so that $z \gtreqless 0 \Rightarrow N^1 \gtreqless N^0$, since $x^1 > 0$ by the assumption that $(N^1, P^1) \neq (N^0, P^0)$. Q.E.D.

Figure 2 illustrates these results. If the term z does not change its sign for all $N \in [\alpha_3, \alpha_2]$ and all $P \leq P^0$, then condition (b) in Proposition 2 becomes necessary as well as sufficient.

The previous analysis emphasized the important role of the function $z(N, P, x)$, which therefore merits additional interpretation. Since z is not dimension-free, the following dimensionless index is defined.

$$I = I(N, P, x) = 1 - \frac{z}{(1-h) f_2 G} = \frac{(1-k)}{(1-h)} \cdot \frac{F}{G} \cdot \frac{g_2}{f_2}. \tag{26}$$

Note that $z \gtreqless 0 \Leftrightarrow I \gtreqless 1$ and that $I > 0$ by (3), (6), (7), and (14). The index $I(N, P, x)$ will be referred to as an *index of impairment*, since it measures the extent to which the predator, as a natural control of the pest, is impaired by the application of the (chemical) control x. This can be seen as follows. The value of $(1-k)/(1-h)$ is the immediate survival rate of the pest relative to that of the predator. If the pest is relatively less susceptible to the control, the index will be higher, ceteris paribus. The terms G and F describe the marginal effect of pesticides applied in time period t on N_{t+1} and P_{t+1}, respectively [Eq. (14)]. If the marginal effect of pesticide on the following period's predator population (relative to the effect on the pest) is stronger, then the absolute value of F is higher than that of G, which implies a higher index I, ceteris paribus. The efficiency of the predator in suppressing the pest is reflected by g_2, while f_2 measures the recovery capacity of the predator after pesticide application through reduced intraspecies competition. As both are negative, the higher the absolute value of the first, and the lower the absolute value of the second, the higher the impairment index will be, ceteris paribus. The size of the impairment index can thus indicate whether introduction of new control may lead to a higher ss pest population ($I > 1$)

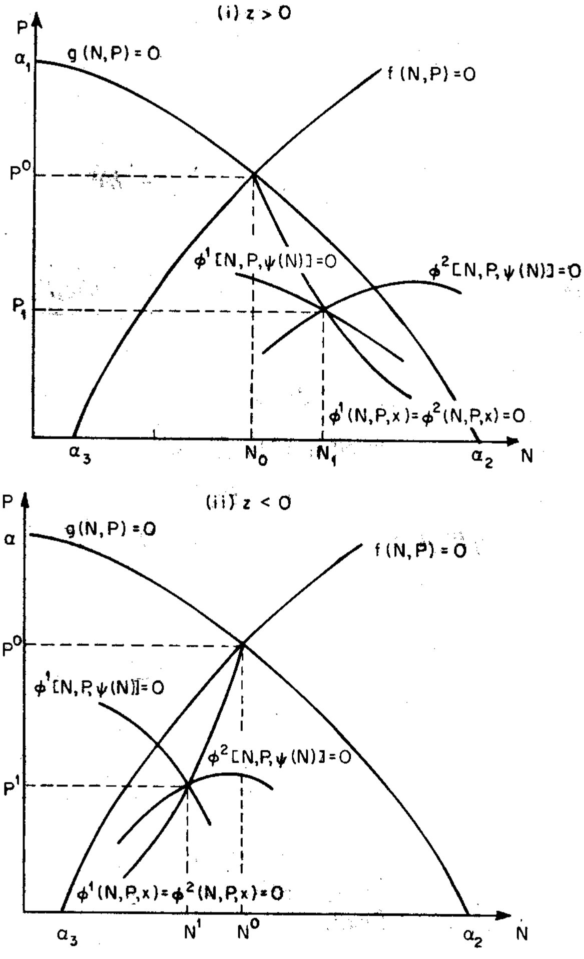

FIGURE 2

or not. The possibility of rising pest resistance to the control is not discussed in the present work (see [13]); however, if pest resistance to the control develops over time, its survival rate $(1 - k)$ increases and the impairment index rises. This will lead to a higher ss pest population if such exists.

Proposition 2 provides conditions that support the findings of entomologists that pest populations in areas where pesticides are introduced have frequently exceeded their prespraying level after some initial success [5, Chap. 1; 25, p. 47].

The possible increase in the pest level as a result of the control application does not necessarily imply that net gain is reduced in the long run; however, such a situation is possible, as will be shown in the following analysis.

Define

$$L^1(N) = \min_x L(N, X) = \delta(N/n)\{1 - \tilde{k}[\psi(N)]\} + C[\psi(N)]. \tag{27}$$

The term L^1 is a monotonically increasing and concave function, by (17), (18), and (19). Comparing the losses at the two equilibrium points, $(N^0, P^0, 0)$ and $[N^1, P^1,$

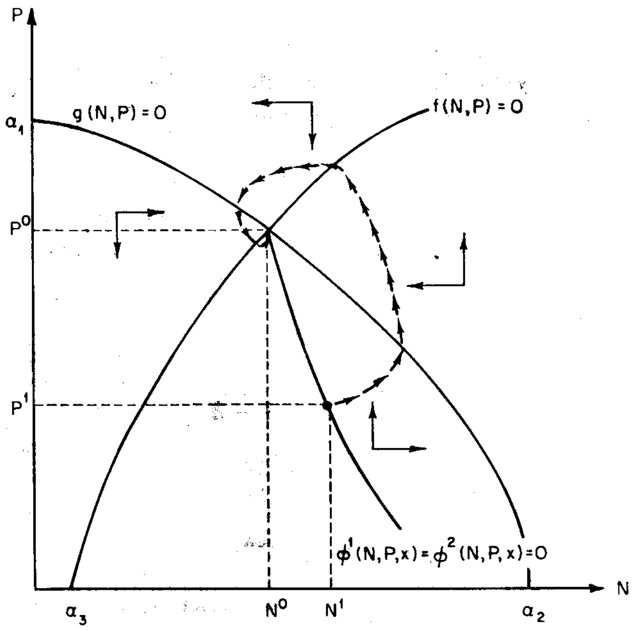

FIGURE 3

$\psi(N^1)]$, the net gain from introducing control for the region as a whole is

$$\delta N^0 - \delta N^1\{1 - k[\psi(N^1)]\} - C[\delta(N^1)]$$
$$= \delta(N^0 - N^1) + \{\delta N^1 k[\psi(N^1)] - C[\psi(N^1)]\}. \quad (28)$$

The second term on the r.h.s. is the total gain of control at N^1 and is positive if control is to be used. Therefore, $N^1 < N^0$ is a sufficient condition for improvement. However, when $N^1 > N^0$, the net gain may be positive or negative.

Note that, though long-run net gain resulting from myopic behavior may be negative, in the short run losses are reduced. This results from a monotonically increasing loss function, $L^1(N)$, and the fact that, immediately following the introduction of control, the pest population falls. Suppose that at the present time $N^1 > N^0$ and the losses at N_1 are greater than those of N^0, i.e., $L^1(N^1) > \delta N^0$. One may be tempted to suggest that the government should overcome this shortsightedness by banning pesticide use. Figure 3 illustrates the time path of N and P if such a decision is made. This path necessarily goes through higher levels of pest populations, and the losses incurred in that period of time may overweight the later gains. The best strategy, starting from any given state, should internalize the externalities previously described, which may be obtained through centralized decision making.

5. CENTRALIZED CONTROL

A central decision agency will take into consideration the environmental cost as well as the population dynamics. Assuming a positive discount rate (r) reflecting time preference, the problem is then to find an optimal time path x_t that

$$\min_{x_t(t) \geq 0} \sum_{t=0}^{\infty} \beta^t\{\delta N_t[1 - k(x_t)] + C(x_t) + E(R_t)\}, \quad (29)$$

where $\beta = (1 + r)^{-1}$ subject to (10), (12), and (13) and given values of the initial conditions, N_{t_0}, P_{t_0}, and R_{t_0}. Assuming that at least one optimal time path exists, some of its properties are analyzed by using the discrete time Pontryagin maximum principle. The following necessary conditions must hold:[13]

$$\delta N k_x - C_x - \beta[\lambda_{t+1} G_t N_t + \mu_{t+1} F_t P_t - \eta_{t+1} n] \leq 0, \tag{30}$$

where strict inequality implies $x_t = 0$; G_t and F_t denote that all arguments of the functions G and F are evaluated at their t period values; and λ_t, μ_t, and η_t are the costate variables of N_t, P_t, and R_t, respectively.

$$\Delta\lambda_t = \lambda_{t+1} - \lambda_t = \delta(1 - k) - \beta\lambda_{t+1}\phi_t^1 + \beta\lambda_{t+1}[r - Ng_1 \cdot (1 - k)^2] \\ - \beta\mu_{t+1}P_t(1 - h)(1 - k)f_1, \tag{31}$$

$$\Delta\mu_t = \mu_{t+1} - \mu_t = - \beta\mu_{t+1}\phi_t^2 + \beta\mu_{t+1}[r - Pf_2(1 - h)^2] \\ - \beta\lambda_{t+1}N(1 - h)(1 - k)g_2, \tag{32}$$

$$\Delta\eta_t = \eta_{t+1} - \eta_t = E_{R_t} + \beta\eta_{t+1}(r + \theta). \tag{33}$$

In addition, Eqs. (10), (12), and (13) must be satisfied, together with the appropriate transversality conditions.

The costate variables, λ_{t+1} and μ_{t+1}, are interpreted as shadow prices of pest and predator populations, respectively. Each measures the effect of an incremental change in the respective population (at time t) on future benefits. The shadow price η_{t+1} is interpreted in the same way as the value of an incremental change in the residue. It is therefore expected that λ_t and η_t will be nonpositive, while μ_t is nonnegative.

Equation (30) is similar to the myopic decision rule (17) except for the expression in parentheses:

$$y_t = \beta[\lambda_{t+1} G_t N_t + \mu_{t+1} P_t F_t - n\eta_{t+1}]. \tag{34}$$

Observe that GN_t and FP_t are the marginal decrements of the pest and predator population stocks, while n is the marginal increment of the residue in the environment as a result of a unit increase in the control x. The value of y_t, therefore, represents an imputed net cost term associated with the marginal control unit that measures future losses (or benefits) resulting from the decision to apply a unit of control at time t. The sign of y is likely to be positive if the environmental costs of the control and its effect on the predator are relatively high; however, it might be negative, in which case the gain from reducing the pest population more than compensates for the above cost. As pointed out by Hueth and Regev [13], the term y may be referred to as "user cost," adopted from the literature on natural resources [3, p. 583; 4, p. 206; 26, p. 190]. In the present context, y is the current value of future gains foregone by the decision to apply a unit of control today, which is analogous to the definition given by Scott [22, p. 34].

Although the signs of λ_t, μ_t, and η_t along the optimal path are not known, they are negative, positive, and negative, respectively, at the ss. To see that, assume $\Delta\lambda = \Delta\mu = \Delta\eta = 0$ and solve for λ^*, μ^*, and η^* (* denotes optimal ss values) from (31) to (33),

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[13] The problem can be equivalently treated by a nonlinear programming approach; see, e.g., [3]. The assumptions made so far do not guarantee sufficiency of these conditions. Sufficiency would be established if we assumed that the maximized Hamiltonian were concave in N, P, and R. This assumption implies certain restrictions on second and cross-partial derivatives of the maximized Hamiltonian with respect to N, P, and R.

using (3) and recalling that $\phi^1 = \phi^2 = 0$ at ss.

$$\lambda^* = -\,\delta(1-k)[r-(1-h)^2P\cdot f_2]/\beta\nabla < 0, \tag{35}$$

$$\mu^* = -\,\delta(1-k)^2\cdot N\cdot(1-h)g_2/\beta\nabla > 0, \tag{36}$$

$$\eta^* = -\,E_R/\beta(r+\theta) < 0, \tag{37}$$

where all variables are evaluated at ss and

$$\nabla = [r-N(1-k)^2g_1][r-P\cdot(1-h)^2f_2]-PN(1-h)^2(1-k)^2g_2 > 0.$$

Using (35) to (37) in (34), the ss user cost is

$$y^* = \frac{\delta N(1-k)G}{\nabla}\left[\frac{(1-h)}{G}Pz-r\right]+\frac{nE_R}{(r+\theta)}, \tag{38}$$

where z is defined in (21). Using (26), it can be seen that a high impairment index ($I > 1 - r/(1-h)^2Pf_2$) implies positive user cost, while if I and the marginal environmental cost (E_R) are sufficiently small, then the user cost will be negative. The decision rule (30) thus equates (when $x > 0$) the marginal benefits (δNk_x) with the marginal cost where the latter is composed of the immediate cost (C_x) and the user cost (y).

The following proposition establishes sufficient conditions for the relative location of a myopic ss and the centralized ss solution. Define

$$\begin{aligned}(N^*,P^*,x^*) &= \text{interior ss satisfying (12), (13), and (30) to (33)}\\ &\quad\ \text{with } \Delta N = \Delta P = \Delta R = \Delta\lambda = \Delta\mu = \Delta\eta = 0,\\ (N^1,P^1,x^1) &= \text{interior ss satisfying (12) and (13)}\\ &\quad\ \text{with } \Delta N = \Delta P = 0 \text{ and (17).}\end{aligned}$$

For any given constant $\hat{y}$ such that

$$\delta Nk_x - C_x - \hat{y} = 0, \tag{39}$$

x can be solved as a function of N and $\hat{y}$:

$$x = \xi(N,\hat{y}),\quad \xi_N = \delta k_x/(C_{xx}-\delta Nk_{xx}) = \psi_N > 0,\quad \xi_{\hat{y}} = -\,\xi_N/\delta k_x < 0. \tag{40}$$

Define the system

$$\begin{aligned}\phi^3 \equiv g(N\cdot\{1-k[\xi(N,\hat{y})]\}, P\cdot\{1-h[\xi(N,\hat{y})\}\})\cdot\{1-k[\xi(N,\hat{y})]\}&\\ -k[\xi(N,\hat{y})] = 0,&\quad (41)\\ \phi^4 \equiv f(N\cdot\{1-k[\xi(N,\hat{y})]\}, P\cdot\{1-h[\xi(N,\hat{y})\}\})\cdot\{1-h[\xi(N,\hat{y})]\}&\\ -h[\xi(N,\hat{y})] = 0,&\end{aligned}$$

and define the sets **A** and **B**:

$$\begin{aligned}\mathbf{A} &= \{z(N,P,x)\,|\,x=\xi(N,\hat{y}),\quad N \text{ and } P \text{ satisfying (41)},\quad \forall\,\hat{y}\in\mathcal{R}\},\\ \mathbf{B} &= \{v(N,P,x)\,|\,x=\xi(N,\hat{y}),\quad N \text{ and } P \text{ satisfying (41)},\quad \forall\,\hat{y}\in\mathcal{R}\},\end{aligned} \tag{42}$$

where z and v are defined by (21) and (22) and $\mathcal{R}$ is the set of $\hat{y}$ values for which (39) holds.

PROPOSITION 3. *Suppose* $z < v\ \forall\, z \in \mathbf{A} \neq \{0\}$, $v \in \mathbf{B} \neq \{0\}$, $y^* \in \mathcal{R}$; *then there exist two ss solutions*, (N^1, P^1, x^1) *and* (N^*, P^*, x^*), *defined above, and they are unique, stable,* *and satisfy*

(*i*) $x^* < x^1, P^* > P^1, N^* > N^1,$ *if* $z < 0$ $\forall z \in A, y^* > 0;$

(*ii*) $x^* < x^1, P^* > P^1, N^* < N^1,$ *if* $0 < z$ $\forall z \in A, y^* > 0;$

(*iii*) $x^* > x^1, P^* < P^1, N^* < N^1,$ *if* $z < 0$ $\forall z \in A, y^* < 0;$

(*iv*) $x^* > x^1, P^* < P^1, N^* > N^1,$ *if* $0 < z$ $\forall z \in A, y^* < 0.$

For proof see the Appendix. The proposition is illustrated by the four diagrams in Figure 4.

The proposition states that positive (negative) user cost implies that, under centralized decision making, the ss level of the control will be lower (higher) than that of the myopic decision rule, implying a lower (higher) accumulated residue. This result is intuitively straightforward, but the sign of the ss user cost (which in many

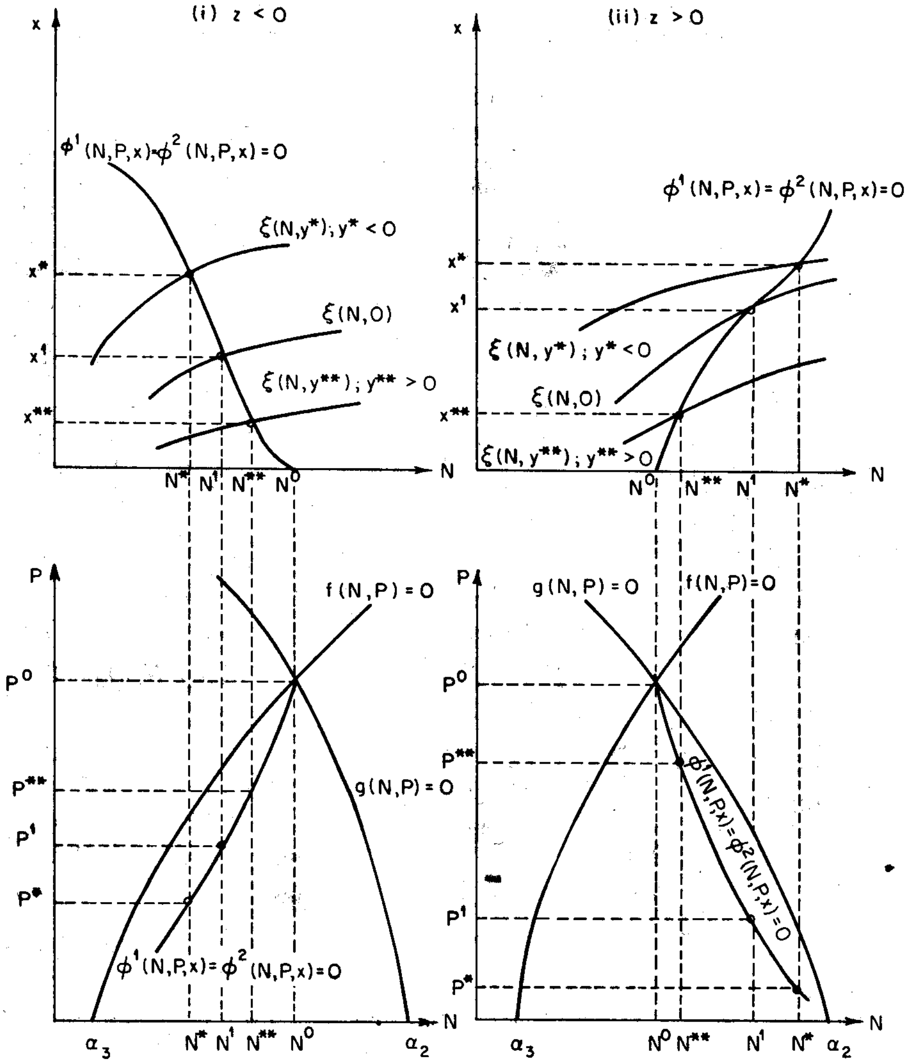

FIGURE 4

other natural resource problems is positive) is not known in the current problem, in which both beneficial and harmful resources are involved. However, this model enables one to make inferences about the sign of the user cost by using information about the impairment index,[14] discount rate, and environmental cost associated with the residue. By (38), it may be stated that $z \geq rG/(1 - h)P^*$ is a sufficient condition for a positive ss user cost. When the environmental costs are negligible, the sign of

$$\{[(1 - h)P^*z/G] - r\}$$

directly determines the sign of y^*. In this case $z < 0$ is a sufficient condition for a negative user cost. In case of a zero discount rate,[15] and negligible residue effect, the sign of z solely determines the sign of the user cost.

To compare the natural ss and the centralized ss, Propositions 2 and 3 are combined, concluding: If the sign of z does not change, centralized management will maintain a higher pest population if $z > 0$, and lower if $z < 0$.

6. SUMMARY AND IMPLICATIONS

The problem of pest control has been tackled in a context of an ecosystem that consists of prey–predator populations with human interaction through pesticide application. The purpose of the control is reduction of pest population, but some undesired by-products are reduction of predator population and contamination of the environment. The untapped biological equilibrium is compared with a decentralized solution of the pest control problem (which for most pests can be regarded as an approximation to the current situation). This comparison emphasized the following points. (a) An index of impairment ($I \geq 0$) is defined and can be used to determine the extent to which the pesticide application impairs the controllability of the pest by its natural enemies (I is estimable from field and laboratory data); (b) if $I < 1$ throughout the relevant range, then the equilibrium resulting from pesticide use will result in lower pest levels when compared to natural equilibrium, while $I > 1$ implies higher ss pest levels; and (c) in the latter case losses may increase under myopic behavior. However, a total ban on pesticides in this case may lead to disastrous results since the pest population will grow even higher (before reaching the lower natural equilibrium). This emphasizes the need for integrated control by which laboratory-reared predators would be introduced into the system. The present model can be extended to include a biological control that enhances the growth of the predator or inhibits the pest growth.

Both pest and predator are regarded as common property resources (harmful and beneficial). The stock externalities inherent in the decentralized solution lead to myopic decision rules that are not optimal from society's viewpoint. An additional external effect for some pesticides is pollution of the environment by pesticide residue. A centralized control solution incorporates the population dynamics and environmental effects. The optimal decision rule then equates marginal benefits with monetary marginal cost of control plus the user cost. The three elements comprising the user cost involve the effect of the marginal unit of control on the stocks of pest, predator, and residue. The larger the effect of the control on the predator and the higher the residue's marginal cost, the higher is the user cost. A positive user cost implies a reduction of pesticide use as a result of implementing centralized decision making. If, however, for a specific pest there are no predators and pesticide use has no residue

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[14] Supra, Section 4.

[15] Mäler [17, pp. 61, 62] presents arguments for a zero discount rate in natural resources.

effect, the user cost will be negative, implying that the decentralized solution results in insufficient pesticide use as compared with the optimal.

It has been shown that, in the absence of environmental costs, the impairment index together with the discount rate and the ss predator population determines whether the user cost will be positive or negative. A sufficiently high discount rate and a low predator population will lead to a negative user cost. A sufficiently low impairment index yields the same result.

Centralized control may be infeasible in many countries. The model, however, supplies the basic analytical tools that may help to institute a policy of tax (or subsidy) that could achieve the same results as the centralized control. If the growth functions of the populations and the environmental cost can be estimated, then an ad valorem tax (or subsidy) equal to the user cost on pesticide use would lead the pesticide users to adopt the optimal policy. Although the information necessary for adopting optimal pest control policies is expensive to obtain, it may be worth paying for in our world of growing needs for food, health, and clean environment.

APPENDIX

Proof of Proposition 3

(a) Since the sets **A** and **B** are assumed nonempty, existence trivially holds.

(b) To prove uniqueness, note that the solution is obtained at the intersection of the two curves [in the (N, P) space], $\phi^3 = 0$ and $\phi^4 = 0$ [defined in (41)]. The curve $\phi^4 = 0$ intersects the curve $\phi^3 = 0$ from below if and only if $z < v$ [see the discussion of stability following (20), and note that $\psi_N = \xi_N$]. Therefore, the intersection must be unique for a given $\hat{y}$.

(c) Local stability requires that, for any given $\hat{y}$,

$$N \cdot [g_1(1-k)^2 - G\xi_N] + P(1-h)^2 f_2 < 0,$$

which holds by (3), (6), (7), (14), and (40); and

$$(1-h)^2[g_1(1-k)^2 - \xi_N G]f_2 - (1-h)(1-k)[(1-h)(1-k)f_1 - \xi_N F]g_2 = (1-h)(v-z)\xi_N > 0,$$

which holds by (6), (7), (14), (40), and the assumption of the proposition that $z < v$.

(d) Differentiate the system (41) and, since $z < v$, then

$$\frac{dN}{d\hat{y}} = \frac{-z}{\delta k_x(v-z)} \gtrless 0 \quad \text{if} \quad z \gtrless 0,$$

$$\frac{dP}{d\hat{y}} = \frac{(1-k)[(1-k)g_1 F - (1-h)f_1 G]}{\delta k_x(1-h)(v-z)} > 0,$$

$$\frac{dx}{d\hat{y}} = \frac{d\xi(N, \hat{y})}{d\hat{y}} = \xi_N \frac{dN}{d\hat{y}} + \xi_v = \xi_v \left[1 - \delta k_x \frac{dN}{d\hat{y}}\right] = \xi_v \cdot \frac{v}{(v-z)} < 0$$

Since the sign of z does not change for all z in **A** (by assumption), then the signs of these derivatives hold for finite changes in $\hat{y}$. The results (i)–(iv) follow immediately.

Q.E.D.

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