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A METHOD FOR IMPROVING CHEAPLY THE TIME RESPONSE OF PRESSURE-TRANSDUCER TENSIOMETER SYSTEMS

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ABSTRACT

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If several tensiometer cups are successively scanned by a single pressure transducer, then if the time constant of one tensiometer is too large, the time for a total scan may become unacceptably long. The effective in situ time constant of each tensiometer can be decreased by connecting to the cup a deformable cell, of which a mercury manometer or a Bourdon pressure gauge are readily available direct-reading examples. Theory is presented to show that provided this cell is disconnected immediately after the transducer has been exposed to it, a useful shortening of the time constant can be obtained by using a Bourdon gauge, but not a mercury manometer.

INTRODUCTION

Some of the advantages gained in using electrical pressure transducers instead of mercury manometers in tensiometer systems may be lost if one transducer is used to sense sequentially several tensiometer cups, as in the systems described, for example, by Rice (1969), Burt (1978), Lee-Williams (1978), and Blackwell and Elsworth (1980). Tensiometers require a finite time to equilibrate with the soil-water pressure, and the transducer must be connected to a tensiometer cup throughout this time if it is to record the pressure correctly. It should be remembered too that although the time response constant of such a transducer tensiometer may be less than a second when measured in free water, it may be considerably longer in soil (see for example, Towner, 1981). Thus if each of say twenty tensiometers requires 2 min for equilibration, it will not be possible to make measurements at any given tensiometer position more often than every 40 min, and this might be too infrequent. One solution for fully exploiting the rapid response characteristics of pressure transducers is to use one pressure transducer per tensiometer, but this is very expensive. Another solution, proposed by M. Goss (private communication, 1981), and explored theoretically in this paper, is to have a cheaper less sensitive pressure measuring device permanently connected to the tensiometer cup, which thus has time to equilibrate before the transducer is switched to it; the transducer thus effectively samples and records the water pressure in this less sensitive gauge.

THEORY

Consider a tensiometer cup, C, connected to two pressure measuring devices (hereafter referred to as "gauges") T_1 and T_2 as shown in Fig. 1; X_1

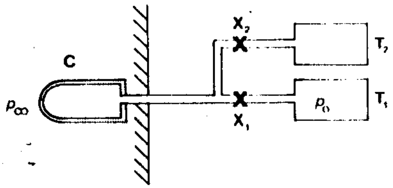

Fig. 1 Diagram of the tensiometer system. C, cup; X_1 and X_2, stopcocks; T_1 and T_2, pressure gauges (e.g. pressure transducer, mercury manometer, Bourdon gauge).

and X_2 are stopcocks. Gauges T_1 and T_2 have sensitivities S_1 and S_2, respectively, where the sensitivity is defined as the change in fluid pressure in the gauge for unit change in volume of fluid in the gauge (Richards and Neal, 1937). Initially, let X_1 be closed and X_2 open, and let gauge T_2 come into equilibrium with the soil-water pressure, p_∞. Then on opening X_1 there will be an instantaneous sharing of pressures between T_1 and T_2 (assuming that the resistance of the connecting tubes is negligible), and the instantaneous pressure p_i will be given by:

$$p_i = \frac{S_1 P_\infty + S_2 p_0}{S_1 + S_2} = \frac{\alpha p_\infty + p_0}{\alpha + 1} \tag{1}$$

where p_0 is the initial pressure in gauge T_1, and $\alpha = S_1/S_2$. Thereafter the gauge pressure p will take time to change from p_i to p_∞, effectively responding as a tensiometer with an in situ time response constant $\Upsilon = \tau_1 + \tau_2$ (where τ_1 and τ_2 are the in situ time constants of T_1 and T_2, respectively). Hence the gauge pressure p at time t is given by

$$\frac{p - p_\infty}{p_i - p_\infty} = \exp\,(-t/\Upsilon)$$

(see Klute and Gardner, 1962). Substituting for p_i from eq. (1), and noting that $\tau_2/\tau_1 = S_1/S_2$, and hence $\Upsilon \approx (1 + \alpha)\tau_1$, we get:

$$\frac{p - p_\infty}{p_0 - p_\infty} = \frac{1}{1 + \alpha} \exp\,(-t/(1+\alpha)\tau_1) \tag{2}$$

The corresponding expression for a single gauge with time constant τ_1 is:

$$\frac{p - p_\infty}{p_0 - p_\infty} = \exp(-t/\tau_1) \quad (3)$$

Hence eqs. (2) and (3) show that at early times the combined system is closer to equilibrium than the single gauge, but at later times the reverse is true (see also Fig. 2, where eqs. (2) and (3) have been plotted for α = 10 and

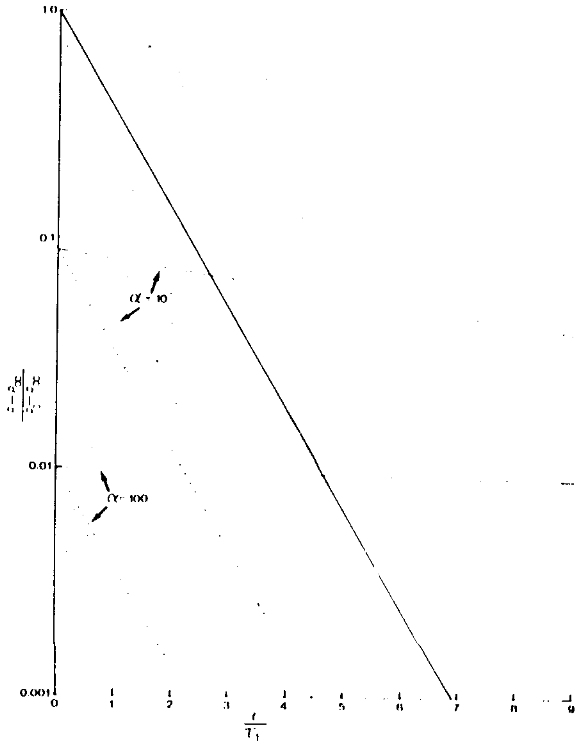

Fig. 2. Tensiometer pressure p plotted as a function of time t, both expressed in dimensionless form. ———, eq. (3); — — —, eq. (2); . . . . , eq. (4). Note that $(p-p_\infty)/(p_0-p_\infty)$ = 0.001 corresponds to 99.9% completion of equilibration.

100). Thus the initial benefit obtained when the pressure changed instantaneously from p_0 to p_i has been lost subsequently because the time constant of the combined tensiometers is greater than that of the single one. This method is not therefore very useful.

However, if X_2 is closed immediately after X_1 is opened, the system response constant is that of T_1, viz. τ_1, and the corresponding time response function is

$$\frac{p - p_\infty}{p_0 - p_\infty} = \frac{1}{1+\alpha} \exp(-t/\tau_1) \tag{4}$$

Thus eq. (4) shows that the benefit of the initial change is now retained for the whole time (see also Fig. 2, where eq. (4) has been plotted for $\alpha = 10$ and 100).

PRACTICABILITY

Equations (1) and (4) provide the theoretical basis for two methods, and the practability of each of these is now examined; eq. (2) does not lead to a useful method. In order to be able to make any useful assessments regarding the practical exploitation of eqs. (1) and (4), it is necessary to take a particular example, but one that is considered representative. We therefore assume that we wish to scan twelve pressures in 30 min, of which two are datum pressures; i.e. there are 150 s available for sampling and recording each tensiometer cup. We will use a pressure transducer for T_1 with the characteristics given in Table I, joined to a cup embedded in a clay soil of

TABLE I

Characteristics of typical pressure gauges

Gauge type	S (MPa m^{-3})	α	Instrument[a] τ (s)	In situ[b] τ (s)
Pressure transducer	100×10^4	–	1.0	50
Bourdon guage	10×10^4	10	10	500
3 mm mercury manometer	1.9×10^4	52	52	2600
3 mm water manometer	0.14×10^4	714	714	36000

[a] For a tensiometer cup with a cup conductance $C = 10^{-6}$ m^3 s^{-1} MPa^{-1}.
[b] Estimated values for soils of hydraulic conductivity of about 10^{-9} m s^{-1}.

hydraulic conductivity of about 10^{-9} m s^{-1}, so that the instrument time constant of 1 s is increased to about 50 s in the soil. We will examine the feasibility of using either a mercury manometer or a Bourdon gauge for T_2 with characteristics as listed in Table I.

Method 1 — eq. (1)

From eq. (1), $p_i \to p_\infty$ as $\alpha \to \infty$, but practically p_i may be regarded as equal to p_∞ for a sufficiently large value of α. However, pressures will be propagated from tensiometer to tensiometer via the scanning switch during the scanning sequence, so that the magnitude of the allowable error is less than might at first have been considered reasonable. (See Appendix I for the theory of the propagation of pressures and for some numerical examples). Fig. 3 shows the relation between α, p_0/p_∞ and p_i/p_∞ for values of p_i/p_∞

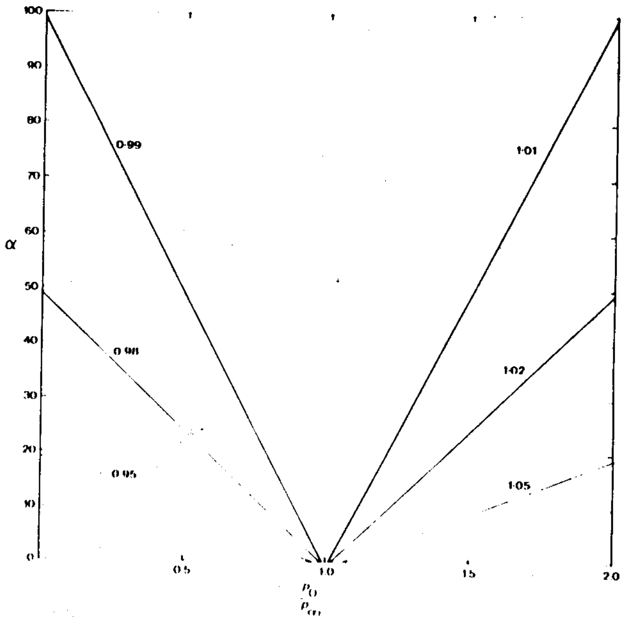

Fig. 3. Nomogram relating α, p_0/p_∞ and p_i/p_∞. The numbers on the curves are values of p_i/p_∞.

close to unity (i.e. to equilibrium). In a scanning system, p_0 will be equal to the previous tensiometer pressure (provided the hydraulic switch does not introduce pressure pulses during its rotation).

Before T_2 is connected to T_1, T_2 must have equilibrated with the soil-water pressure p_∞ in 30 min or less (i.e. within the total scan time for twelve pressures), implying an in situ time constant $\lesssim$ 360 s. Only the Bourdon gauge, for which $\alpha = 10$, satisfies this condition (see Table I). From Fig. 3, we see that for $\alpha = 10$ and provided $1.1 > p_0/p_\infty > 0.9$, then $p_i/p_\infty \approx 0.99$, in others words, provided that p_0 is close to p_∞, the transducer records instantaneously a value which is within 1 per cent equal to the soil-water pressure. If the soil-water pressure changes appreciably during the 30 min period, more severe conditions must be applied, and it is unlikely that this method could prove practical, since T_2 would never be in equilibrium with the soil-water pressure.

Method 2 — eq. (4)

When it is not possible to assume that $p_i = p_\infty$, then the sensitivity of T_2 must be such that the time required for T_1 to equilibrate is equal to (or less than) the sampling time; for our example, $t \leqslant 150$ s. Thus taking τ_1 as 50 s and $(p\text{-}p_\infty)/(p_0\text{-}p_\infty) \leqslant 0.01$ (corresponding to $\geqslant$ 99% completion of equilibration), then eq. (4) requires $\alpha \geqslant 4$. The Bourdon gauge satisfies this requirement (see Table I), and does equilibrate with the soil-water pressure within the total scan time of 30 min (see under Method 1). This would not be true if the soil-water pressure was changing rapidly, but nevertheless T_1 would be closer to the true value in Method 2 than it would acting alone.

CONCLUSIONS

It is not possible to conclude categorically that Method 1 or Method 2 will necessarily achieve the desired result of decreasing the effective time response constant of a pressure transducer tensiometer system; each must be examined on its own merits. The paper has however shown that for a particular system Method 1 is of dubious value, but that Method 2 can provide the desired improvement. The paper has not discussed the mechanical aspects. The extra switching required in Method 2 would not be difficult to achieve in practice, but it would add extra expense. Moreover, the additional pressure gauges do not in fact have to be standard dial-reading Bourdon gauges, since they only have to act as reservoirs, but the latter would require further development.

Finally, it is recommended that if it is neccesary to design a system to operate as Method 2, then the original sampling strategy should be reconsidered. It might for instance be better to increase the sampling time to ensure equilibration, thus obtaining better data although less frequently; or to make measurements at fewer positions and use a separate pressure transducer with each cup.

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APPENDIX I

Propagation of pressures during scanning

Let a scanning pressure transducer sample n constant pressures $p_1, p_2, p_3, \ldots, p_r, \ldots, p_n$ in turn, allowing the same sampling time Δt per pressure reading. The time constant is assumed the same for each tensiometer. If the initial pressure in the transducer at the time of sampling is assumed to be that established by the previous cup, and has a value p_0 before sampling commences, then the pressure p'_r recorded by the transducer at the end of the sampling time Δt is given by eq. (3) as:

$$p'_r = p_r + (p'_{r-1} - p_r) \exp(-\Delta t/\tau)$$

Hence:

$$\begin{aligned}
p'_1 &= p_1 + (p_0 - p_1) \exp(-\Delta t/\tau) \\
p'_2 &= p_2 + (p'_1 - p_2) \exp(-\Delta t/\tau) \\
&= p_2 + (p_1 + (p_0 - p_1) \exp(-\Delta'/\tau) - p_2) \exp(-\Delta t/\tau) \\
&= p_2 + (p_1 - p_2) \exp(-\Delta t/\tau) + (p_0 - p_1) \exp(-2\Delta t/\tau)
\end{aligned}$$

$$p'_n = p_n + \sum_{r=1}^{n} (p_{n-r} - p_{n-r+1}) \exp(-r\,\Delta t/\tau)$$

Thus the expression for p'_n includes contributions from $p_{n-1}, p_{n-2}, p_{n-3}, \ldots, p_0$, but with decreasing influence.

In a scanning system, the sampling cycle returns to p_1 after p_n, with $p_0 = p'_n$; and the whole cycle thus repeats with a new set of pressure readings, but these become constant after a few cycles; in effect, the initial pressure p_0 is dissipated.

Some numerical examples have been collected in Tables II, III, IV and V to illustrate the magnitudes of the errors than can occur when insufficient time is allowed for equilibration. It is not possible to cover all the permutations that may occur in practice. Therefore, the examples have been selected to draw attention to some points that should be born in mind, rather than to simulate actual systems.

Thus the tables show that the ordering of sampling should be designed, if possible, so that each pressure is followed by a pressure of about the same magnitude. For example, Table II, which could represent measurements made in order along a horizontal flow system, shows that all values are within 1% of the equilibrium value at a sampling time of $\Delta\tau = 5\tau$ except for $p'_r/p_r = 1.0$ where the error is 6%. However, if the sequence of sampling is simply altered to that given in Table III, the largest error is reduced to 1%.

Tables IV and V demonstrate the effect of assuming that one pressure always comes into instantaneous equilibrium with the sampled value as would occur for example when the corresponding tensiometer is replaced by an open-water reservoir maintained at a constant level. Typical field measuring systems would generally include two such datam levels for monitoring the calibration. Table IV shows that the effect of holding the first pressure value of the sequence used in Table I as the reference value, whilst Table V shows that for holding the last value. The influence of the reference level is complicated,

TABLE II

Tensiometer readings, from the sequential sampling of ten cups exposed to constant pressures given respectively by p_r'/p_1 = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (relative units), for different sampling times Δt (expressed relative to the response time constant)

$\frac{\Delta t}{\tau}$	$\frac{p_r'}{p_1}$									
∞	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
5	1.06	1.99	2.99	3.99	4.99	5.99	6.99	7.99	8.99	9.99
4	1.16	1.98	2.98	3.98	4.98	5.98	6.98	7.98	8.98	9.98
3	1.45	1.97	2.95	3.95	4.95	5.95	6.95	7.95	8.95	9.95
2	2.20	2.03	2.87	3.85	4.84	5.84	6.84	7.84	8.84	9.84
1	4.10	2.77	2.92	3.60	4.48	5.44	6.43	7.42	8.42	9.42

TABLE III

As Table II, but with the pressures sampled in the order 2, 4, 6, 8, 10, 9, 7, 5, 3, 1

$\frac{\Delta t}{\tau}$	$\frac{p_r}{p_{10}}$									
∞	2.00	4.00	6.00	8.00	10.00	9.00	7.00	5.00	3.00	1.00
5	1.99	3.99	5.99	7.99	9.99	9.01	7.01	5.01	3.01	1.01
4	1.98	3.96	5.96	7.96	9.96	9.02	7.04	5.04	3.04	1.04
3	1.96	3.90	5.90	7.90	9.90	9.05	7.10	5.10	3.11	1.10
2	1.91	3.72	5.69	7.69	9.69	9.09	7.28	5.31	3.31	1.31
1	2.05	3.28	5.00	6.90	8.86	8.95	7.72	6.00	4.10	2.14

TABLE IV

As Table II, but with the tensiometer gauge equilibrating each time with (relative) pressure 1, as would occur with a direct connection to a datum pressure

$\frac{\Delta t}{\tau}$	$\frac{p_r'}{p_1}$									
∞	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
5	1.00	1.99	2.99	3.99	4.99	5.99	6.99	7.99	8.99	9.99
4	1.00	1.98	2.98	3.98	4.98	5.98	6.98	7.98	8.98	9.98
3	1.00	1.95	2.95	3.95	4.95	5.95	6.95	7.95	8.95	9.95
2	1.00	1.87	2.85	3.84	4.84	5.84	6.84	7.84	8.84	9.84
1	1.00	1.63	2.50	3.45	4.43	5.42	6.42	7.42	8.42	9.42

TABLE V

As Table IV, but with the tensiometer gauge attaining equilibrium each time with the (relative) pressure 10

$\frac{\Delta t}{\tau}$	$\frac{p'_r}{p_1}$									
∞	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
5	1.06	1.99	2.99	3.99	4.99	5.99	6.99	7.99	8.99	10.00
4	1.16	1.98	2.98	3.98	4.98	5.98	6.98	7.98	8.98	10.00
3	1.45	1.97	2.95	3.95	4.95	5.95	6.95	7.95	8.95	10.00
2	2.22	2.03	2.87	3.85	4.84	5.84	6.84	7.84	8.84	10.00
1	4.31	2.85	2.95	3.61	4.49	5.44	6.43	7.42	8.42	10.00

being beneficial for some values and detrimental for others in the sequence, although these effects are only apparent in the Tables at the shortest sampling times. Table V amplifies the danger of following a high value ($p'_r/p_1 = 10$) by a low value ($p'_r/p_1 = 1.0$).

The tables generally show that the effect of altering the order of sampling is to redistribute the potential errors. Thus when the sampling time is less than ideal, it may be possible to exercise some control over the distribution of errors by a judicious arrangement of the sample sequence.